UNIT 9: Real-life and algebraic linear graphs

SPECIFICATION REFERENCES

N13 use standard units of mass, length, time, money and other measures (including standard compound measures) using decimal quantities where appropriate
A7 where appropriate, interpret simple expressions as functions with inputs and outputs
A8 work with coordinates in all four quadrants
A9 plot graphs of equations that correspond to straight-line graphs in the coordinate plane; …
A10 identify and interpret gradients and intercepts of linear functions graphically and algebraically
A12 Recognise, sketch and interpret graphs of linear functions …
A14 plot and interpret … graphs of non-standard functions in real contexts, to find approximate solutions to problems such as simple kinematic problems involving distance, speed and acceleration
A17 solve linear equations in one unknown algebraically (including those with the unknown on both sides of the equation); find approximate solutions using a graph
R1 change freely between related standard units (e.g. time, length, area, volume/capacity, mass) and compound units (e.g. speed, rates of pay, prices, density, pressure) in numerical and algebraic contexts
R11 use compound units such as speed, … unit pricing, …
R14 interpret the gradient of a straight line graph as a rate of change; recognise and interpret graphs that illustrate direct and inverse proportion
G11 solve geometrical problems on coordinate axes
G14 use standard units of measure and related concepts (length, area, volume/capacity, mass, time, money, etc.)

PRIOR KNOWLEDGE

Students should be able to plot coordinates and read scales

Students should be able to substitute into a formula.

KEYWORDS

Tier 2

Linear, graph, distance, time

Tier 3

Quadrant, gradient, intercept, parallel

9a. Real-life graphs (N13, A7, A8, A9, A10, A14, R1, R11, R14, G11, G14)	Teaching time 8–10 hours

OBJECTIVES

By the end of the sub-unit, students should be able to:

- Use input/output diagrams;
- Use axes and coordinates to specify points in all four quadrants in 2D;
- Identify points with given coordinates and coordinates of a given point in all four quadrants;
- Find the coordinates of points identified by geometrical information in 2D (all four quadrants);
- Find the coordinates of the midpoint of a line segment;
- Draw, label and scale axes;
- Read values from straight-line graphs for real-life situations;
- Draw straight line graphs for real-life situations, including ready reckoner graphs, conversion graphs, fuel bills graphs, fixed charge and cost per unit;
- Draw distance–time graphs and velocity–time graphs;
- Work out time intervals for graph scales;
- Interpret distance–time graphs, and calculate: the speed of individual sections, total distance and total time;
- Interpret information presented in a range of linear and non-linear graphs;
- Interpret graphs with negative values on axes;
- Find the gradient of a straight line from real-life graphs;
- Interpret gradient as the rate of change in distance–time and speed–time graphs, graphs of containers filling and emptying, and unit price graphs.

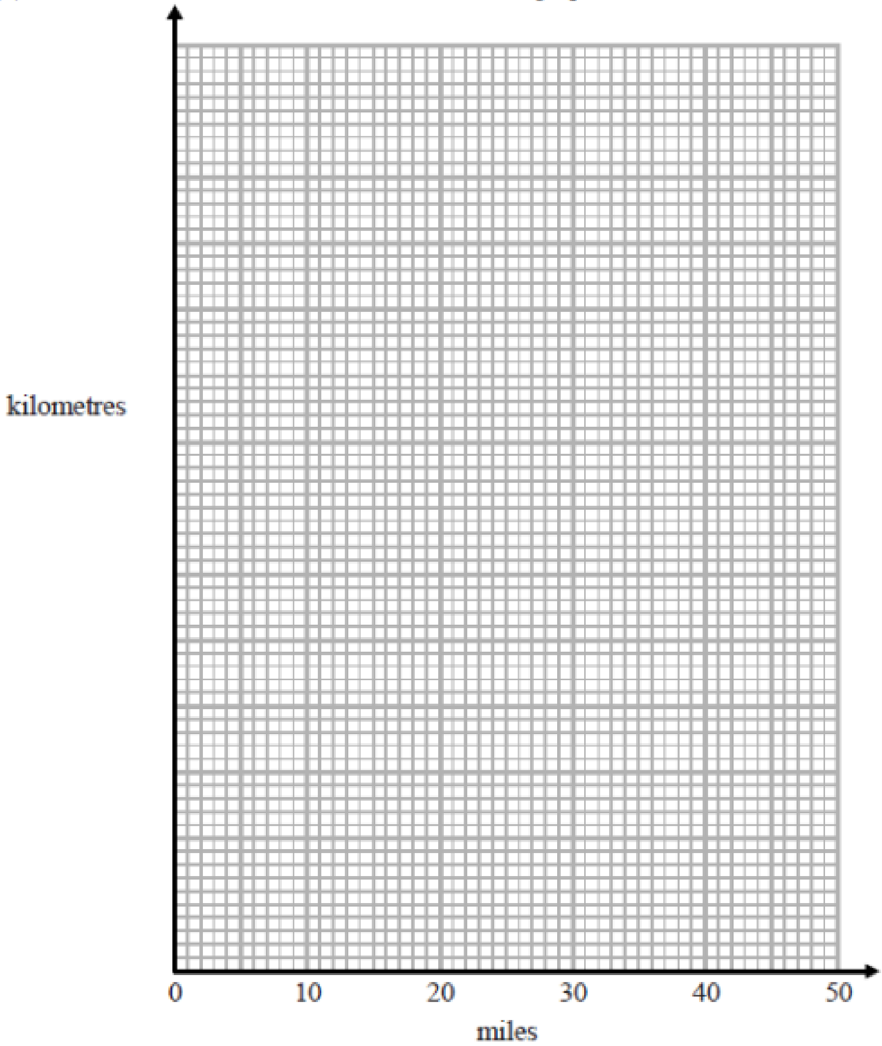
POSSIBLE SUCCESS CRITERIA/EXAM QUESTIONS

Interpret a description of a journey into a distance–time or speed–time graph.

You can use the information in the table to convert between kilometres and miles.

miles	0	5	20	40
kilometres	0	8	32	64

(*a*) Use this information to draw a conversion graph.



(3)

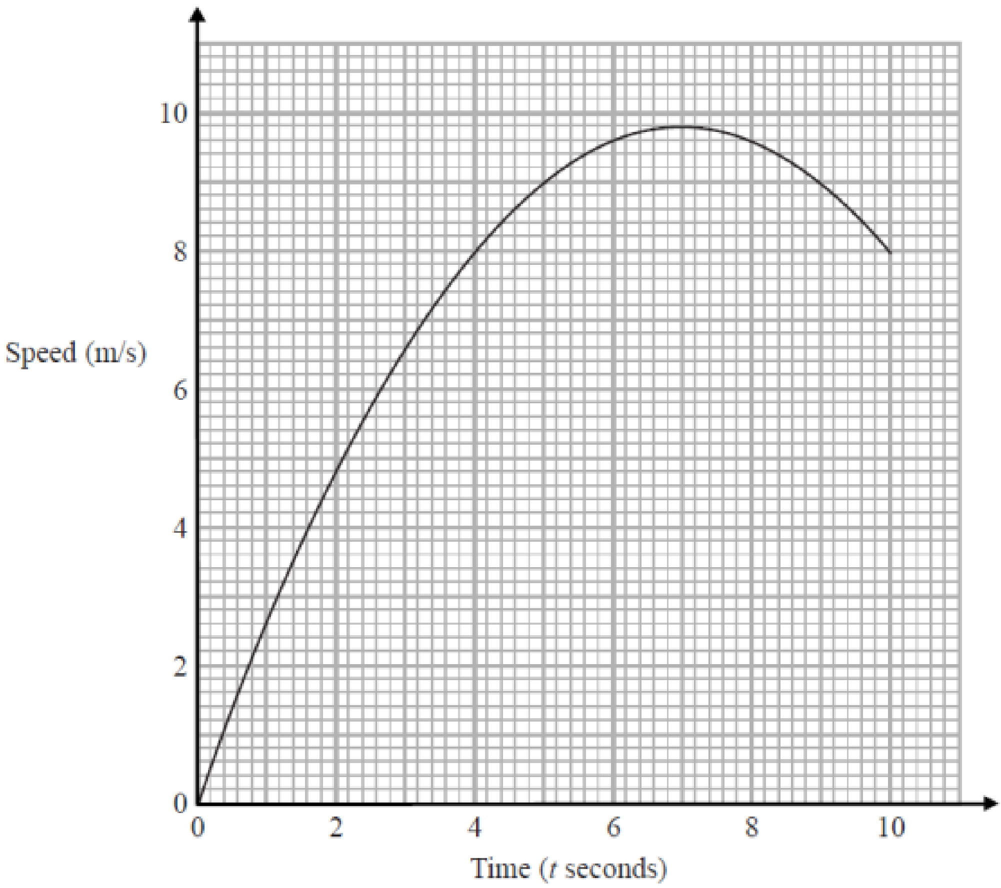
(*b*) Which is further, 20 kilometres or 15 miles?
You must show how you got your answer.

(2)

(Total 5 marks)

Karol ran in a race.

The graph shows her speed, in metres per second, t seconds after the start of the race.



(*a*) Write down Karol's speed 3 seconds after the start of the race.

(1)

(*b*) Write down Karol's greatest speed.

(1)

There were two times when Karol's speed was 9 m/s.

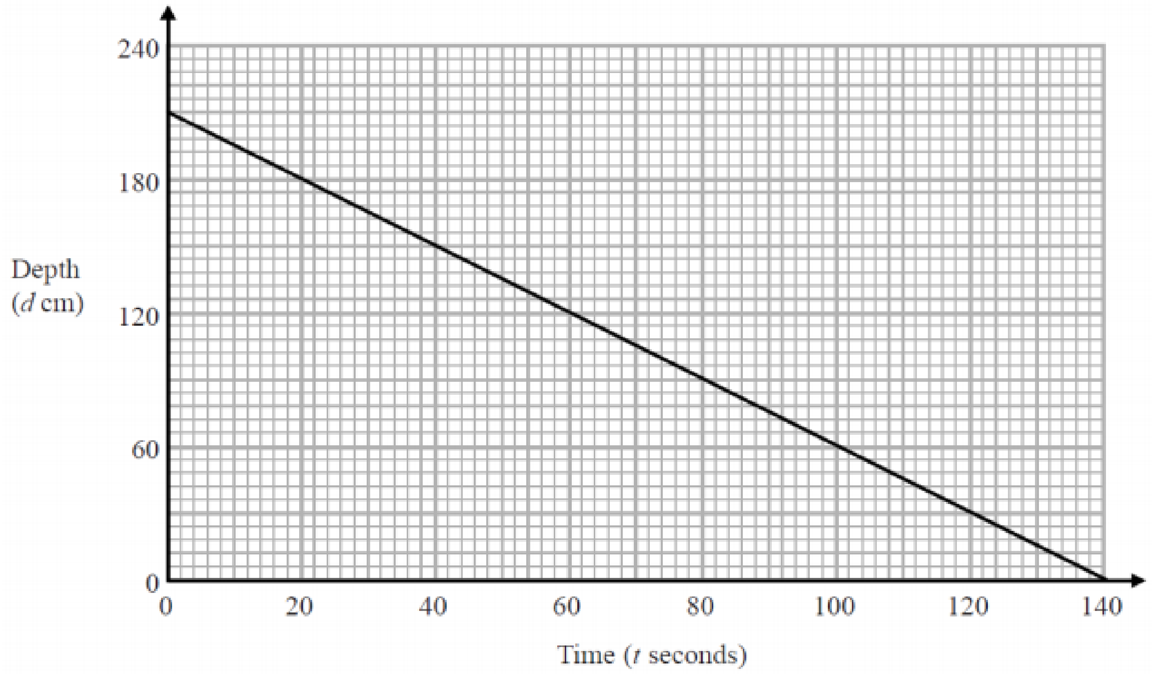
(*c*) Write down these two times.

(1)

(Total 3 marks)

Specimen Papers Set 2, Paper 2F qu.10 (A14 –AO2)

The graph shows the depth, d cm, of water in a tank after t seconds.



(a) Find the gradient of this graph.

(2)

(b) Explain what this gradient represents.

(1)

(Total 3 marks)

Specimen Papers Set 1, Paper 2H qu.10 (R14 – AO1/AO2)

OPPORTUNITIES FOR REASONING/PROBLEM SOLVING

Students should be able to decide what the scales on any axis should be to be able to draw a correct graph.

Conversion graphs can be used to provide opportunities for students to justify which distance is further, or whether or not certain items can be purchase in different currencies.

COMMON MISCONCEPTIONS

With distance–time graphs, students struggle to understand that the perpendicular distance from the x-axis represents distance.

NOTES

Clear presentation of axes is important.

Ensure that you include questions that include axes with negative values to represent, for example, time before present time, temperature or depth below sea level.

Careful annotation should be encouraged: it is good practice to get the students to check that they understand the increments on the axes.

Use standard units of measurement to draw conversion graphs.

Use various measures in distance–time and velocity–time graphs, including miles, kilometres, seconds, and hours.

9b. Straight-line graphs (A7, A9, A10, A12, A17)	**Teaching time** 5–7 hours

OBJECTIVES

- By the end of the sub-unit, students should be able to:
- Use function machines to find coordinates (i.e. given the input x, find the output y);
- Plot and draw graphs of $y = a$, $x = a$, $y = x$ and $y = -x$;
- Recognise straight-line graphs parallel to the axes;
- Recognise that equations of the form $y = mx + c$ correspond to straight-line graphs in the coordinate plane;
- Plot and draw graphs of straight lines of the form $y = mx + c$ using a table of values;
- Sketch a graph of a linear function, using the gradient and y-intercept;
- Identify and interpret gradient from an equation $y = mx + c$;
- Identify parallel lines from their equations;
- Plot and draw graphs of straight lines in the form $ax + by = c$;
- Find the equation of a straight line from a graph;
- Find the equation of the line through one point with a given gradient;
- Find approximate solutions to a linear equation from a graph;

POSSIBLE SUCCESS CRITERIA/EXAM QUESTIONS

Plot and draw the graph for $y = 2x - 4$.

Which of these lines are parallel: $y = 2x + 3$, $y = 5x + 3$, $y = 2x - 9$, $2y = 4x - 8$

Here are the equations of four straight lines.

Line A	$y = 2x + 4$
Line B	$2y = x + 4$
Line C	$2x + 2y = 4$
Line D	$2x - y = 4$

Two of these lines are parallel.

Write down the two parallel lines.

(Total 1 mark)

Specimen Papers Set 1, Paper 3F qu.27 / 3H qu.7 (A9 – AO2)

OPPORTUNITIES FOR REASONING/PROBLEM SOLVING

Students should be able to decide what the scales on any axis should in order to draw a correct graph.

COMMON MISCONCEPTIONS

When not given a table of values, students rarely see the relationship between the coordinate axes.

NOTES

Emphasise the importance of drawing a table of values when not given one.

Values for a table should be taken from the x-axis.

UNIT 10: Transformations

SPECIFICATION REFERENCES

R6 express a multiplicative relationship between two quantities as a ratio or a fraction
R12 … make links to similarity … and scale factors
G1 use conventional terms and notations: points, lines, vertices, edges, planes, parallel lines, perpendicular lines, right angles, polygons, regular polygons and polygons with reflection and/or rotation symmetries; …
G7 identify, describe and construct congruent and similar shapes, including on coordinate axes, by considering rotation, reflection, translation and enlargement (including fractional scale factors)
G24 describe translations as 2D vectors

PRIOR KNOWLEDGE

Students should recall basic shapes.
Students should be able to plot points in all four quadrants.
Students should have an understanding of the concept of rotation.
Students should be able to draw and recognise lines parallel to axes and $y = x$, $y = -x$.

KEYWORDS

Tier 2

Transform, translation, describe, similar, combination

Tier 3

Rotation, reflection, enlargement, scale factor, mirror line, centre of rotation, centre of enlargement, column vector, vector, congruent, angle, direction, coordinate

10a. Transformations I: rotations and translations (G1, G7, G24)	Teaching time 5–7 hours

OBJECTIVES

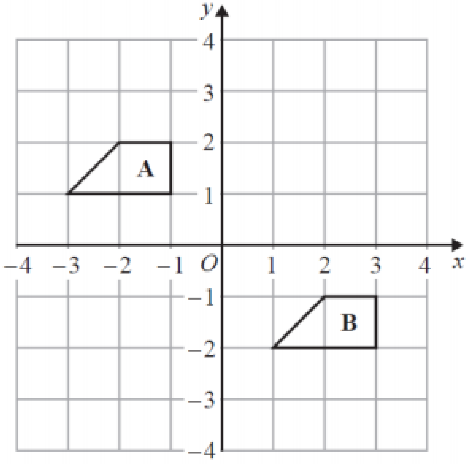
By the end of the sub-unit, students should be able to:

- Identify congruent shapes by eye;
- Understand clockwise and anticlockwise;
- Understand that rotations are specified by a centre, an angle and a direction of rotation;
- Find the centre of rotation, angle and direction of rotation and describe rotations;
- Describe a rotation fully using the angle, direction of turn, and centre;
- Rotate a shape about the origin or any other point on a coordinate grid;
- Draw the position of a shape after rotation about a centre (not on a coordinate grid);
- Identify correct rotations from a choice of diagrams;
- Understand that translations are specified by a distance and direction using a vector;
- Translate a given shape by a vector;
- Describe and transform 2D shapes using single translations on a coordinate grid;
- Use column vectors to describe translations;
- Understand that distances and angles are preserved under rotations and translations, so that any figure is congruent under either of these transformations.

POSSIBLE SUCCESS CRITERIA/EXAM QUESTIONS

Understand that translations are specified by a distance and direction (using a vector).

Describe and transform a given shape by either a rotation or a translation.

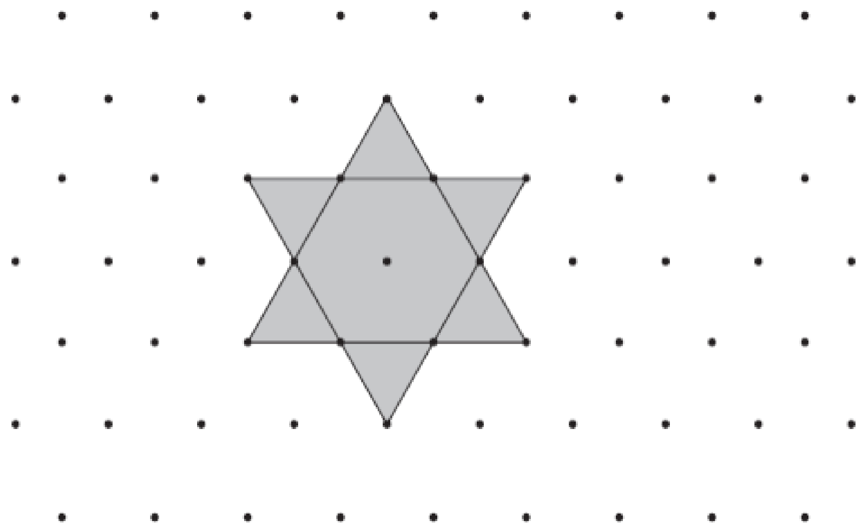


Describe the single transformation that maps shape **A** onto shape **B**.

(Total 2 marks)

Specimen Papers Set 2, Paper 2F qu.22 / 2H qu.1 (G24, G7 – AO2)

Here is a star shape.



The star shape is made from a regular hexagon and six congruent equilateral triangles.
The area of the star shape is 96 cm^2.
Work out the area of the regular hexagon.

(Total 2 marks)

New SAMs Paper 3F qu.12 (G7, G4 – AO1/AO3)

COMMON MISCONCEPTIONS

The directions on a column vector often get mixed up.

Student need to understand that the 'units of movement' are those on the axes, and care needs to be taken to check the scale.

Correct language must be used: students often use 'turn' rather than 'rotate'.

NOTES

Emphasise the need to describe the transformations fully, and if asked to describe a 'single' transformation they should not include two types.

Include rotations with the centre of rotation inside the shape.

Use trial and error with tracing paper to find the centre of rotation.

It is essential that the students check the increments on the coordinate grid when translating shapes.

10b. Transformations II: reflections and enlargements	**Teaching time**
(R6, R12, G1, G7)	7–9 hours

OBJECTIVES

By the end of the sub-unit, students should be able to:

- Understand that reflections are specified by a mirror line;
- Identify correct reflections from a choice of diagrams;
- Understand that reflections are specified by a mirror line;
- Identify the equation of a line of symmetry;
- Transform 2D shapes using single reflections (including those not on coordinate grids) with vertical, horizontal and diagonal mirror lines;
- Describe reflections on a coordinate grid;
- Scale a shape on a grid (without a centre specified);
- Understand that an enlargement is specified by a centre and a scale factor;
- Enlarge a given shape using (0, 0) as the centre of enlargement, and enlarge shapes with a centre other than (0, 0);
- Find the centre of enlargement by drawing;
- Describe and transform 2D shapes using enlargements by:
    - a positive integer scale factor;
    - a fractional scale factor;
- Identify the scale factor of an enlargement of a shape as the ratio of the lengths of two corresponding sides, simple integer scale factors, or simple fractions;
- Understand that distances and angles are preserved under reflections, so that any figure is congruent under this transformation;
- Understand that similar shapes are enlargements of each other and angles are preserved – define similar in this unit;

POSSIBLE SUCCESS CRITERIA

Describe and transform a given shape by a reflection.

Convince me the scale factor is, for example, 2.5.

OPPORTUNITIES FOR REASONING/PROBLEM SOLVING

Students should be given the opportunity to explore the effect of reflecting in two parallel mirror lines and combining transformations.

NOTES

Emphasise the need to describe the transformations fully and if asked to describe a 'single' transformation they should not include two types.

Students may need reminding about how to find the equations of straight lines, including those parallel to the axes.

When reflecting shapes, the students must include mirror lines on or through original shapes.

As an extension, consider reflections with the mirror line through the shape and enlargements with the centre of enlargement inside the shape.

NB enlargement using negative scale factors is not included.