UNIT 12: Right-angled triangles: Pythagoras and trigonometry	**Teaching Time** **5–7 hours**

SPECIFICATION REFERENCES

N7 calculate with roots, and with integer indices
N15 round numbers and measures to an appropriate degree of accuracy (e.g. to a specified number of decimal places or significant figures); …
A4 simplify and manipulate algebraic expressions (including those involving surds) by: collecting like terms, multiplying a single term over a bracket, …
A5 understand and use standard mathematical formulae; …
R12 … make links to similarity (including trigonometric ratios) …
G6 apply angle facts, triangle congruence, similarity and properties of quadrilaterals to conjecture and derive results about angles and sides, including Pythagoras' Theorem and the fact that the base angles of an isosceles triangle are equal, and use known results to obtain simple proofs
G20 know the formulae for: Pythagoras' Theorem $a^2 + b^2 = c^2$ and the trigonometric ratios, sine, cosine and tan; apply them to find angles and lengths in right-angled triangles in two dimensional figures
G21 know the exact values of sin θ and cos θ for θ = 0°, 30°, 45°, 60° and 90°; know the exact value of tan θ for θ = 0°, 30°, 45° and 60°

PRIOR KNOWLEDGE

Students should be able to rearrange simple formulae and equations, as preparation for rearranging trigonometric formulae.

Students should recall basic angle facts.

Students should understand when to leave an answer in surd form.

Students can plot coordinates in all four quadrants and draw axes.

KEYWORDS

Tier 2

Opposite, adjacent, accuracy, elevation, depression

Tier 3

Triangle, right angle, angle, Pythagoras' Theorem, sine, cosine, tan, trigonometry, hypotenuse

OBJECTIVES

By the end of the unit, students should be able to:

- Understand, recall and use Pythagoras' Theorem in 2D, including leaving answers in surd form;
- Given 3 sides of a triangle, justify if it is right-angled or not;
- Calculate the length of the hypotenuse in a right-angled triangle, including decimal lengths and a range of units;
- Find the length of a shorter side in a right-angled triangle;
- Apply Pythagoras' Theorem with a triangle drawn on a coordinate grid;
- Calculate the length of a line segment AB given pairs of points;
- Understand, use and recall the trigonometric ratios sine, cosine and tan, and apply them to find angles and lengths in general triangles in 2D figures;
- Use the trigonometric ratios to solve 2D problems;
- Find angles of elevation and depression;
- Round answers to appropriate degree of accuracy, either to a given number of significant figures or decimal places, or make a sensible decision on rounding in context of question;
- Know the exact values of sin θ and cos θ for θ = 0°, 30°, 45°, 60° and 90°; know the exact value of tan θ for θ = 0°, 30°, 45° and 60°.

POSSIBLE SUCCESS CRITERIA/EXAM QUESTIONS

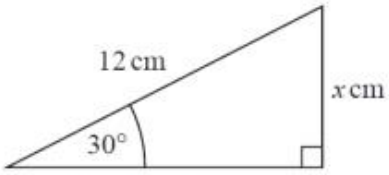
Does 2, 3, 6 give a right angled triangle?

Justify when to use Pythagoras' Theorem and when to use trigonometry.

(*a*) Write down the exact value of cos30°

(1)

(*b*)



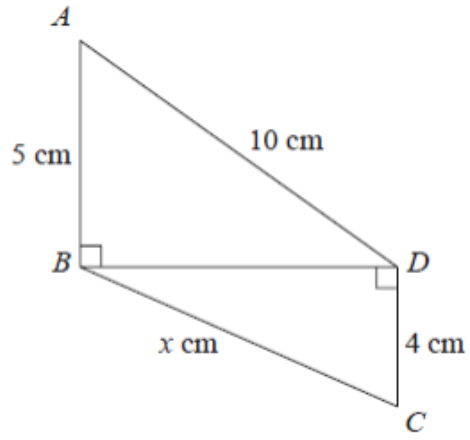
Given that sin 30° = 0.5, work out the value of x.

(2)

(Total 3 marks)

Specimen Papers Set 1, Paper 1F qu.26 / 1H qu.7 (G21, G20 – AO1)

Triangles *ABD* and *BCD* are right-angled triangles.



Work out the value of x.
Give your answer correct to 2 decimal places.

(Total 4 marks)

New SAMs Paper 2F qu.28 / 2H qu.6 (G20 – AO1/AO3)

OPPORTUNITIES FOR REASONING/PROBLEM SOLVING

Combined triangle problems that involve consecutive application of Pythagoras' Theorem or a combination of Pythagoras' Theorem and the trigonometric ratios.

In addition to abstract problems, students should be encouraged to apply Pythagoras' Theorem and/or the trigonometric ratios to real-life scenarios that require them to evaluate whether their answer fulfils certain criteria, e.g. the angle of elevation of 6.5 m ladder cannot exceed 65°. What is the greatest height it can reach?

COMMON MISCONCEPTIONS

Answers may be displayed on a calculator in surd form.

Students forget to square root their final answer or round their answer prematurely.

NOTES

Students may need reminding about surds.

Drawing the squares on the 3 sides will help to illustrate the theorem.

Include examples with triangles drawn in all four quadrants.

Scale drawings are not acceptable.

Calculators need to be in degree mode.

To find in right-angled triangles the exact values of sin θ and cos θ for θ = 0°, 30°, 45°, 60° and 90°, use triangles with angles of 30°, 45° and 60°.

Use a suitable mnemonic to remember SOHCAHTOA.

Use Pythagoras' Theorem and trigonometry together.