**UNIT 13: Probability**

## SPECIFICATION REFERENCES

N5 apply systematic listing strategies
P1 record, describe and analyse the frequency of outcomes of probability experiments using tables and frequency trees
P2 apply ideas of randomness, fairness and equally likely events to calculate expected outcomes of multiple future experiments
P3 relate relative expected frequencies to theoretical probability, using appropriate language and the 0–1 probability scale
P4 apply the property that the probabilities of an exhaustive set of outcomes sum to one; apply the property that the probabilities of an exhaustive set of mutually exclusive events sum to one
P5 understand that empirical unbiased samples tend towards theoretical probability distributions, with increasing sample size
P6 enumerate sets and combinations of sets systematically, using tables, grids, Venn diagrams and tree diagrams
P7 construct theoretical possibility spaces for single and combined experiments with equally likely outcomes and use these to calculate theoretical probabilities
P8 calculate the probability of independent and dependent combined events, including using tree diagrams and other representations, and know the underlying assumptions

## PRIOR KNOWLEDGE

Students should know how to add and multiply fractions and decimals.
Students should have experience of expressing one number as a fraction of another number.

## KEYWORDS

Tier 2

Dependent, independent, conditional, outcomes, fairness

Tier 3

Probability, tree diagrams, sample space, theoretical, relative frequency, experimental

| **13a. Probability I** | **Teaching time** |
|---|---|
| (P1, P2, P3, P4, P6, P7) | 4–6 hours |

**OBJECTIVES**

By the end of the sub-unit, students should be able to:

- Distinguish between events which are impossible, unlikely, even chance, likely, and certain to occur;
- Mark events and/or probabilities on a probability scale of 0 to 1;
- Write probabilities in words or fractions, decimals and percentages;
- Find the probability of an event happening using theoretical probability;
- Use theoretical models to include outcomes using dice, spinners, coins;
- List all outcomes for single events systematically;
- Work out probabilities from frequency tables;
- Work out probabilities from two-way tables;
- Record outcomes of probability experiments in tables;
- Add simple probabilities;
- Identify different mutually exclusive outcomes and know that the sum of the probabilities of all outcomes is 1;
- Using 1 – $p$ as the probability of an event not occurring where $p$ is the probability of the event occurring;
- Find a missing probability from a list or table including algebraic terms.

**POSSIBLE SUCCESS CRITERIA/EXAM QUESTIONS**

Mark events on a probability scale and use the language of probability.

If the probability of outcomes are $x$, $2x$, $4x$, $3x$ calculate $x$.

Calculate the probability of an event from a two-way table or frequency table.

Decide if a coin, spinner or game is fair.

$\mathscr{E} = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$
$A = \{\text{multiples of } 2\}$
$A \cap B = \{2, 6\}$
$A \cup B = \{1, 2, 3, 4, 6, 8, 9, 10\}$

Draw a Venn diagram for this information.

**(Total 4 marks)**

*Specimen Papers Set 1, Paper 3F qu.20 (P6, N4 – AO2)*

There are 25 boys and 32 girls in a club.

$\frac{2}{5}$ of the boys and $\frac{1}{2}$ of the girls walk to the club.

The club leader picks at random a child from the children who walk to the club.

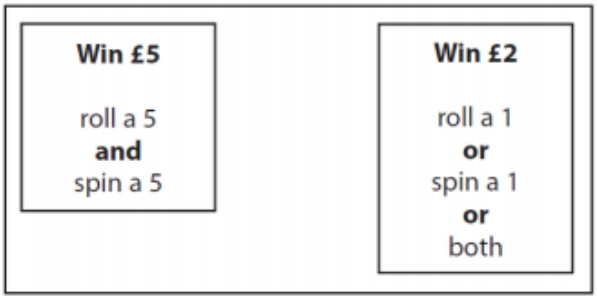
Work out the probability that this child is a boy.

**(Total 3 marks)**

*New SAMs Paper 3F qu.15 (P3, N8 – AO1/AO3)*

David has designed a game.
He uses a fair 6-sided dice and a fair 5-sided spinner.
The dice is numbered 1 to 6.
The spinner is numbered 1 to 5.

Each player rolls the dice once and spins the spinner once.
A player can win £5 or win £2.




David expects 30 people will play his game.
Each person will pay David £1 to play the game.

(*a*) Work out how much profit David can expect to make.

**(4)**

(*b*) Give a reason why David's actual profit may be different to the profit he expects to make.

**(1)**

**(Total 5 marks)**

*Specimen Papers Set 2, Paper 1F qu.21 / 1H qu.3 (P2, P5, P8 – AO1/AO3)*

**OPPORTUNITIES FOR REASONING/PROBLEM SOLVING**

Students should be given the opportunity to justify the probability of events happening or not happening.

**NOTES**

Use this as an opportunity for practical work.

Probabilities written in fraction form should be cancelled to their simplest form.

| **13b. Probability II** | **Teaching time** |
|---|---|
| (N5, P1, P2, P3, P5, P7, P8) | 8–10 hours |

**OBJECTIVES**

By the end of the sub-unit, students should be able to:

- Find the probability of an event happening using relative frequency;
- Estimate the number of times an event will occur, given the probability and the number of trials – for both experimental and theoretical probabilities;
- List all outcomes for combined events systematically;
- Use and draw sample space diagrams;
- Work out probabilities from Venn diagrams to represent real-life situations and also 'abstract' sets of numbers/values;
- Use union and intersection notation;
- Compare experimental data and theoretical probabilities;
- Compare relative frequencies from samples of different sizes;
- Find the probability of successive events, such as several throws of a single dice;
- Use tree diagrams to calculate the probability of two independent events;
- Use tree diagrams to calculate the probability of two dependent events.

**POSSIBLE SUCCESS CRITERIA/EXAM QUESTIONS**

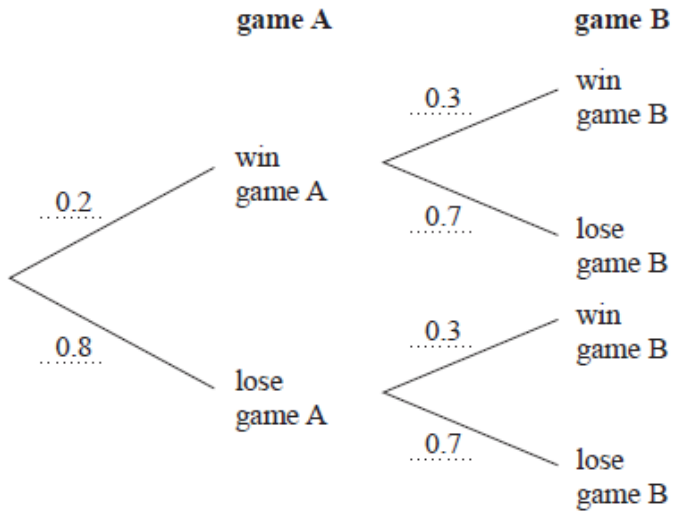
Understand the use of the 0–1 scale to measure probability.

List all the outcomes for an experiment.

Know and apply the fact that the sum of probabilities for all outcomes is 1.

Draw a Venn diagram of students studying French, German or both, and then calculate the probability that a student studies French given that they also study German.

Here is a probability tree diagram.




Work out the probability of winning both games.

**(Total 2 marks)**

*New SAMs Paper 2F qu.29 (P8 – AO1/AO2)*

## OPPORTUNITIES FOR REASONING/PROBLEM SOLVING

Lotteries provides a real life link to probability. Work out the probabilities of winning on different lotteries.

## COMMON MISCONCEPTIONS

Not using fractions or decimals when working with probability trees.

## NOTES

Probability without replacement is best illustrated visually and by initially working out probability 'with' replacement.

Encourage students to work 'across' the branches working out the probability of each successive event. The probability of the combinations of outcomes should = 1.

Emphasise that were an experiment repeated it will usually lead to different outcomes, and that increasing sample size generally leads to better estimates of probability and population characteristics.

Probabilities written in fraction form should be cancelled to their simplest form.

**UNIT 15: Constructions: triangles, nets, plan and elevation, loci, scale drawings and bearings**

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## SPECIFICATION REFERENCES

R2 use scale factors, scale diagrams and maps
G1 use conventional terms and notation: points, lines, vertices, edges, planes, parallel lines, perpendicular lines, right angles, polygons, regular polygons and polygons with reflection and/or rotation symmetries; use the standard conventions for labelling and referring to the sides and angles of triangles; draw diagrams from written description;
G2 use the standard ruler and compass constructions (perpendicular bisector of a line segment, constructing a perpendicular to a given line from/at a given point, bisecting a given angle); use these to construct given figures and solve loci problems; know that the perpendicular distance from a point to a line is the shortest distance to the line
G5 use the basic congruence criteria for triangles (SSS, SAS, ASA, RHS)
G9 identify and apply circle definitions and properties, including: centre, radius, chord, diameter, circumference, tangent, arc, sector and segment
G12 identify properties of the faces, surfaces, edges and vertices of: cubes, cuboids, prisms, cylinders, pyramids, cones and spheres
G13 construct and interpret plans and elevations of 3D shapes
G15 measure line segments and angles in geometric figures, including interpreting maps and scale drawings and use of bearings

## PRIOR KNOWLEDGE

Students should be able to measure and draw lines.

## KEYWORDS

Tier 2

Construct, face, edge, elevation, scale, plan, region, degree, regular, irregular

Tier 3

Circle, arc, sector, vertex, two-dimensional, three-dimensional, solid, congruent, angles, bearing, bisect, perpendicular, loci, map

| **15a. Plans and elevations**<br><br>(G1, G2, G9, G12, G13, G15) | **Teaching time**<br><br>5–7 hours |
|---|---|

**OBJECTIVES**

By the end of the sub-unit, students should be able to:

- Understand clockwise and anticlockwise;
- Draw circles and arcs to a given radius or given the diameter;
- Measure and draw lines, to the nearest mm;
- Measure and draw angles, to the nearest degree;
- Know and use compass directions;
- Draw sketches of 3D solids;
- Know the terms face, edge and vertex;
- Identify and sketch planes of symmetry of 3D solids;
- Make accurate drawings of triangles and other 2D shapes using a ruler and a protractor;
- Construct diagrams of everyday 2D situations involving rectangles, triangles, perpendicular and parallel lines;
- Understand and draw front and side elevations and plans of shapes made from simple solids;
- Given the front and side elevations and the plan of a solid, draw a sketch of the 3D solid.

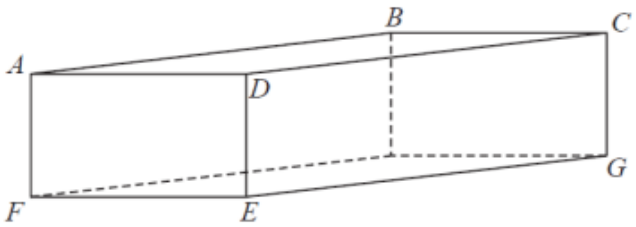
**POSSIBLE SUCCESS CRITERIA/EXAM QUESTIONS**

Be able to estimate the size of given angles.

Convert fluently between metric units of length.

Use bearings in a real-life context to describe the bearing between two towns on a map.

Here is a cuboid.




Some of the vertices are labelled.

(*b*) Shade in the face *CDEG*.

**(1)**

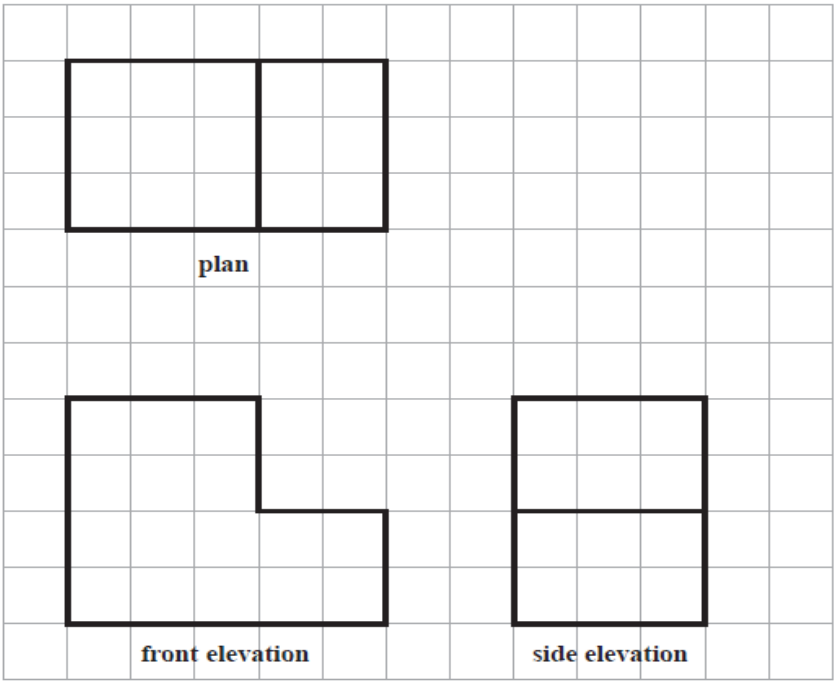
(*c*) How many edges has a cuboid?

**(1)**

**(Total 3 marks)**

*Specimen Papers Set 1, Paper 3F qu.14 (G12, G1 – AO1)*

The plan, front elevation and side elevation of a solid prism are drawn on a centimetre grid.

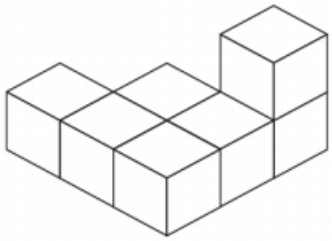



In the space below, draw a sketch of the solid prism.
Write the dimensions of the prism on your sketch.

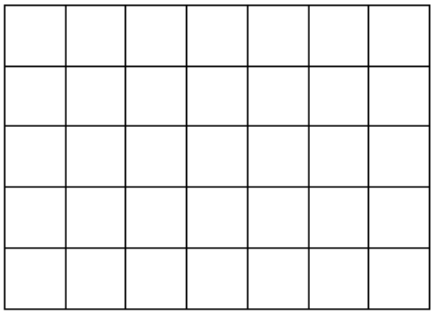
**(Total 2 marks)**

*New SAMs Paper 1F qu.26 / 1H qu.6 (G13 – AO2)*

The diagram represents a solid made from seven centimetre cubes.



On the centimetre grid below, draw a plan of the solid.



**(Total 2 marks)**

*Specimen Papers Set 2, Paper 3F qu.23 / 3H qu.2 (G13 – AO2)*

## OPPORTUNITIES FOR REASONING/PROBLEM SOLVING

Interpreting scale drawings and maps involving lengths that need to be measured (rather than given in the problem).

## COMMON MISCONCEPTIONS

Some pupils may use the wrong scale of a protractor. For example, they measure an obtuse angle as 60° rather than as 120°.

Often 5 sides only are drawn for a cuboid.

## NOTES

This is a very practical topic, and provides opportunities for some hands-on activities.

Drawing 3D shapes in 2D using isometric grids isn't an explicit objective but provides an ideal introduction to the topic and for some students provides the scaffolding needed when drawing 3D solids.

Whilst not an explicit objective, it is useful for students to draw and construct nets and show how they fold to make 3D solids, allowing students to make the link between 3D shapes and their nets. This will enable students to understand that there is often more than one net that can form a 3D shape.

| **15b. Constructions, loci and bearings**<br><br>(R2, G2, G5, G15) | **Teaching time**<br><br>9–11 hours |
|---|---|

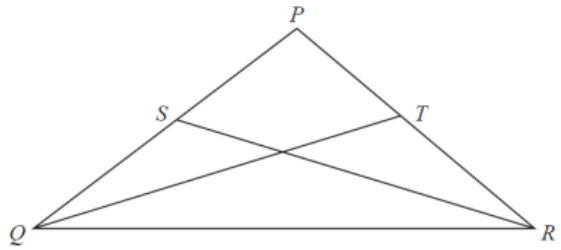
**OBJECTIVES**

By the end of the sub-unit, students should be able to:

- Understand congruence, as two shapes that are the same size and shape;
- Visually identify shapes which are congruent;
- Use straight edge and a pair of compasses to do standard constructions:
  - understand, from the experience of constructing them, that triangles satisfying SSS, SAS, ASA and RHS are unique, but SSA triangles are not;
  - construct the perpendicular bisector of a given line;
  - construct the perpendicular from a point to a line;
  - construct the bisector of a given angle;
  - construct angles of 90°, 45°;
- Draw and construct diagrams from given instructions, including the following:
  - a region bounded by a circle and an intersecting line;
  - a given distance from a point and a given distance from a line;
  - equal distances from two points or two line segments;
  - regions may be defined by 'nearer to' or 'greater than';
- Find and describe regions satisfying a combination of loci;
- Use constructions to solve loci problems (2D only);
- Use and interpret maps and scale drawings;
- Estimate lengths using a scale diagram;
- Make an accurate scale drawing from a diagram;
- Use three-figure bearings to specify direction;
- Mark on a diagram the position of point $B$ given its bearing from point $A$;
- Give a bearing between the points on a map or scaled plan;
- Given the bearing of a point $A$ from point $B$, work out the bearing of $B$ from $A$;
- Use accurate drawing to solve bearings problems;
- Solve locus problems including bearings.

**POSSIBLE SUCCESS CRITERIA/EXAM QUESTIONS**

Sketch the locus of point on a vertex of a rotating shape as it moves along a line, i.e. a point on the circumference or at the centre of a wheel.




$PQ = PR$.
$S$ is the midpoint of $PQ$.
$T$ is the midpoint of $PR$.

Prove triangle $QTR$ is congruent to triangle $RSQ$.

**(Total 3 marks)**

*Specimen Papers Set 1, Paper 1H qu.17 (G5 – AO2)*

The diagram shows a rectangle $ABCD$.


In the space below, use a ruler and a pair of compasses to construct a right-angled triangle equal in area to the area of the rectangle $ABCD$.

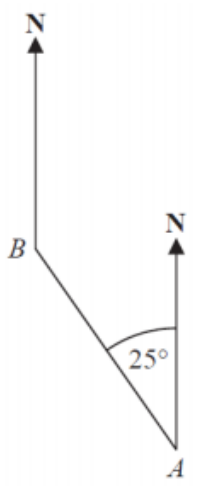
You must show all your construction lines.

The base of the triangle, which is equal in length to the side $CD$, has been drawn for you.


**(Total 3 marks)**

*Original SAMs Paper 3F qu.17 / 3H qu.6 (G2, G16 – AO2)*

The diagram shows the positions of two churches, $A$ and $B$.




Amber says,

"The bearing of church $B$ from church $A$ is 025°"

Amber is wrong.
Explain why.

**(Total 1 mark)**

*Specimen Papers Set 2, Paper 3F qu.18 (G15 – AO2)*

## OPPORTUNITIES FOR REASONING/PROBLEM SOLVING

Link problems with other areas of mathematics, such as the trigonometric ratios and Pythagoras' Theorem.

## COMMON MISCONCEPTIONS

Correct use of a protractor may be an issue.

## NOTES

Drawings should be done in pencil.

Relate loci problems to real-life scenarios, including mobile phone masts and coverage.

Construction lines should not be erased.