UNIT 6: Real-life and algebraic linear graphs, quadratic and cubic graphs, the equation of a circle, plus rates of change and area under graphs made from straight lines

SPECIFICATION REFERENCES

N13 use standard units of mass, length, time, money and other measures (including standard compound measures) using decimal quantities where appropriate

A8 work with coordinates in all four quadrants

A9 plot graphs of equations that correspond to straight-line graphs in the coordinate plane; use the form $y = mx + c$ to identify parallel **and perpendicular lines**; find the equation of the line through two given points, or through one point with a given gradient

A10 identify and interpret gradients and intercepts of linear functions graphically and algebraically

A11 identify and interpret roots, intercepts, turning points of quadratic functions graphically; …

A12 recognise, sketch and interpret graphs of linear functions, quadratic functions, simple cubic functions, the reciprocal function with $x \neq 0$, …

A14 plot and interpret … graphs of non-standard functions in real contexts to find approximate solutions to problems such as simple kinematic problems involving distance, speed and acceleration

A15 **calculate or estimate gradients of graphs and areas under graphs (including quadratic and non-linear graphs) and interpret results in cases such as distance–time graphs, velocity–time graphs … (this does not include calculus)**

A16 **recognise and use the equation of a circle with centre at the origin; find the equation of a tangent to a circle at a given point**

A17 solve linear equations in one unknown … (including those with the unknown on both sides of the equation); find approximate solutions using a graph

A18 solve quadratic equations (**including those that require rearrangement**) algebraically by factorising, **by completing the square and by using the quadratic formula**; find approximate solutions using a graph

R1 change freely between related standard units (e.g. time, length, area, volume/capacity, mass) and compound units (e.g. speed, rates of pay, prices, density, pressure) in numerical and algebraic contexts

R10 solve problems involving direct … proportion, including graphical … representations

R11 use compound units such as speed, … unit pricing, …

R14 … recognise and interpret graphs that illustrate direct and inverse proportion

PRIOR KNOWLEDGE

Students can identify coordinates of given points in the first quadrant or all four quadrants.

Students can use Pythagoras' Theorem and calculate the area of compound shapes.

Students can use and draw conversion graphs for these units.

Students can use function machines and inverse operations.

KEYWORDS

Tier 2
Speed, distance, time, solution, root, approximate, gradient

Tier 3
Coordinate, axes, 3D, Pythagoras, graph, velocity, quadratic, function, linear, circle, cubic, perpendicular, parallel, equation

6a. Graphs: the basics and real-life graphs (N13, A8, A9, A10, A14, A15, R1, R11)	Teaching time 6–8 hours

OBJECTIVES

By the end of the sub-unit, students should be able to:

- Identify and plot points in all four quadrants;
- Draw and interpret straight-line graphs for real-life situations, including ready reckoner graphs, conversion graphs, fuel bills, fixed charge and cost per item;
- Draw distance–time and velocity–time graphs;
- Use graphs to calculate various measures (of individual sections), including: unit price (gradient), average speed, distance, time, acceleration; including using enclosed areas by counting squares or using areas of trapezia, rectangles and triangles;
- Find the coordinates of the midpoint of a line segment with a diagram given and coordinates;
- Find the coordinates of the midpoint of a line segment from coordinates;
- Calculate the length of a line segment given the coordinates of the end points;
- Find the coordinates of points identified by geometrical information.
- Find the equation of the line through two given points.

POSSIBLE SUCCESS CRITERIA/EXAM QUESTIONS

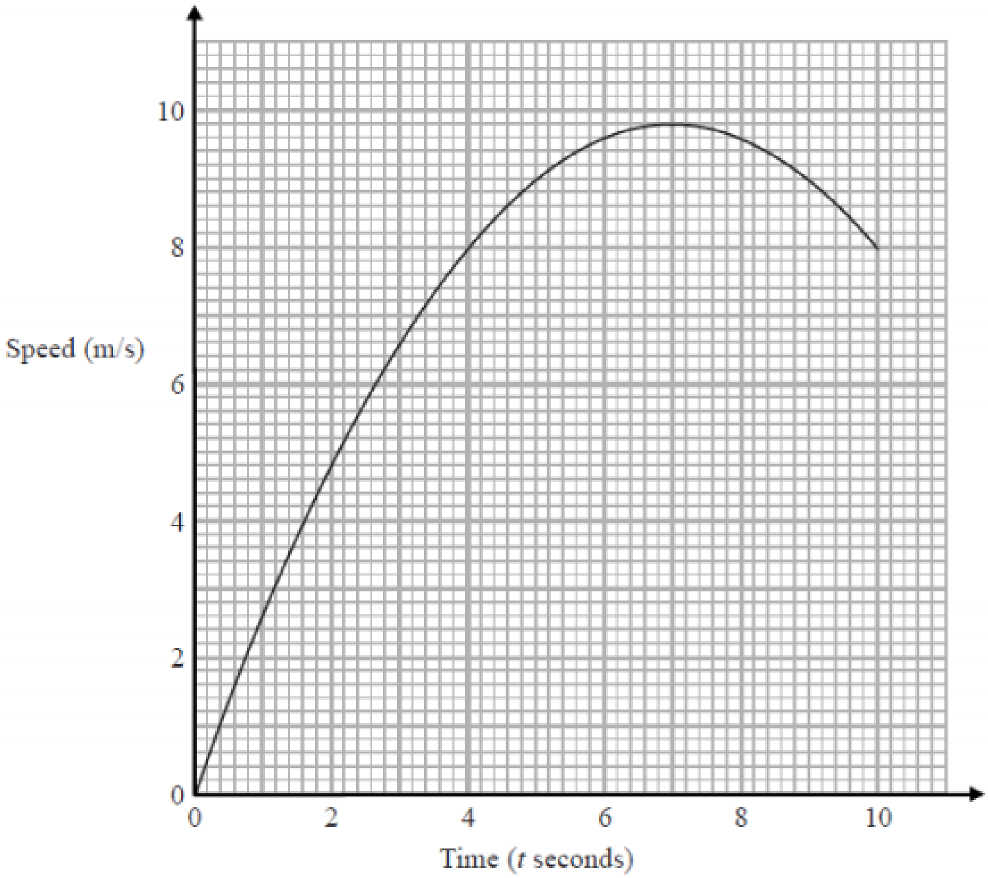
Interpret a description of a journey into a distance–time or speed–time graph.

Calculate various measures given a graph.

Calculate an end point of a line segment given one coordinate and its midpoint.

Karol runs in a race.

The graph shows her speed, in metres per second, t seconds after the start of the race.



(a) Calculate an estimate for the gradient of the graph when $t = 4$
You must show how you get your answer.

(3)

(b) Describe fully what your answer to part (a) represents.

(2)

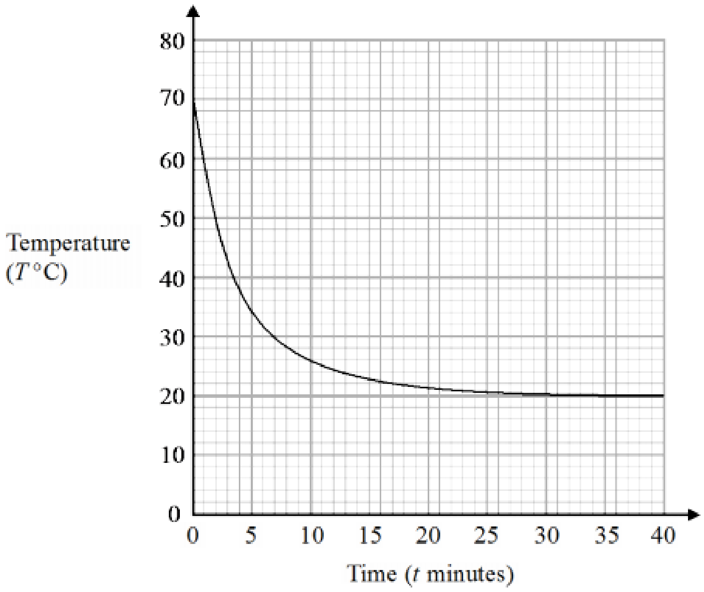
(c) Explain why your answer to part (a) is only an estimate.

(1)

(Total 6 marks)

Specimen Papers Set 2, Paper 2H qu.15 (A15 –AO1/AO2/AO3)

The graph shows the temperature, T °C, of the coffee in a cup at a time t minutes.



(a) Find an estimate for the gradient of the graph at time 5 minutes.

(2)

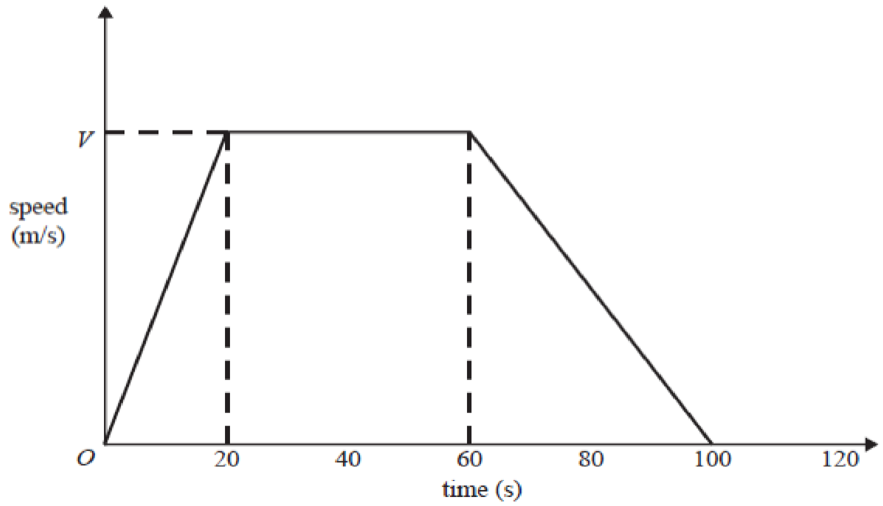
(b) Explain what this gradient represents.

(1)

(Total 3 marks)

Mock Papers Set 3, Paper 1H qu.13 (A15, R15 – AO1/AO2)

Here is a speed-time graph for a car journey.
The journey took 100 seconds.



The car travelled 1.75 km in the 100 seconds.

(a) Work out the value of V.

(3)

(b) Describe the acceleration of the car for each part of this journey.

(2)

(Total 5 marks)

New SAMs Paper 1H qu.21 (A15 – AO1/AO2/AO3)

OPPORTUNITIES FOR REASONING/PROBLEM SOLVING

Speed/distance graphs can provide opportunities for interpreting non-mathematical problems as a sequence of mathematical processes, whilst also requiring students to justify their reasons why one vehicle is faster than another.

Calculating the length of a line segment provides links with other areas of mathematics.

COMMON MISCONCEPTIONS

Where line segments cross the y-axis, finding midpoints and lengths of segments is particularly challenging as students have to deal with negative numbers.

NOTES

Careful annotation should be encouraged: it is good practice to label the axes and check that students understand the scales.

Use various measures in the distance–time and velocity–time graphs, including miles, kilometres, seconds, and hours, and include large numbers in standard form.

Ensure that you include axes with negative values to represent, for example, time before present time, temperature or depth below sea level.

Metric-to-imperial measures are not specifically included in the programme of study, but it is a useful skill and ideal for conversion graphs.

Emphasise that velocity has a direction.

Coordinates in 3D can be used to extend students.

6b. Linear graphs and coordinate geometry (A9, A10, A12, A17, R10, R14)	Teaching time 9–11 hours

OBJECTIVES

By the end of the unit, students should be able to:

- Plot and draw graphs of $y = a$, $x = a$, $y = x$ and $y = -x$, drawing and recognising lines parallel to axes, plus $y = x$ and $y = -x$;
- Identify and interpret the gradient of a line segment;
- Recognise that equations of the form $y = mx + c$ correspond to straight-line graphs in the coordinate plane;
- Identify and interpret the gradient and y-intercept of a linear graph given by equations of the form $y = mx + c$;
- Find the equation of a straight line from a graph in the form $y = mx + c$;
- Plot and draw graphs of straight lines of the form $y = mx + c$ with and without a table of values;
- Sketch a graph of a linear function, using the gradient and y-intercept (i.e. without a table of values);
- Find the equation of the line through one point with a given gradient;
- Identify and interpret gradient from an equation $ax + by = c$;
- Find the equation of a straight line from a graph in the form $ax + by = c$;
- Plot and draw graphs of straight lines in the form $ax + by = c$;
- Interpret and analyse information presented in a range of linear graphs:
  - use gradients to interpret how one variable changes in relation to another;
  - find approximate solutions to a linear equation from a graph;
  - identify direct proportion from a graph;
  - find the equation of a line of best fit (scatter graphs) to model the relationship between quantities;
- Explore the gradients of parallel lines and lines perpendicular to each other;
- Interpret and analyse a straight-line graph and generate equations of lines parallel and perpendicular to the given line;
- Select and use the fact that when $y = mx + c$ is the equation of a straight line, then the gradient of a line parallel to it will have a gradient of m and a line perpendicular to this line will have a gradient of .

POSSIBLE SUCCESS CRITERIA/EXAM QUESTIONS

Find the equation of the line passing through two coordinates by calculating the gradient first.

Understand that the form $y = mx + c$ or $ax + by = c$ represents a straight line.

$A(-2, 1)$, $B(6, 5)$, and $C(4, k)$ are the vertices of a right-angled triangle ABC.
Angle ABC is the right angle.

Find an equation of the line that passes through A and C.
Give your answer in the form $ay + bx = c$ where a, b and c are integers.

(Total 5 marks)
New SAMs Paper 1H qu.25 (A9 – AO1/AO3)

OPPORTUNITIES FOR REASONING/PROBLEM SOLVING

Given an equation of a line provide a counter argument as to whether or not another equation of a line is parallel or perpendicular to the first line.

Decide if lines are parallel or perpendicular without drawing them and provide reasons.

COMMON MISCONCEPTIONS

Students can find visualisation of a question difficult, especially when dealing with gradients resulting from negative coordinates.

NOTES

Encourage students to sketch what information they are given in a question – emphasise that it is a sketch.

Careful annotation should be encouraged – it is good practice to label the axes and check that students understand the scales.

6c. Quadratic, cubic and other graphs	**Teaching time**
(A11, A12, A14, A16, A18)	7–9 hours

OBJECTIVES

By the end of the sub-unit, students should be able to:

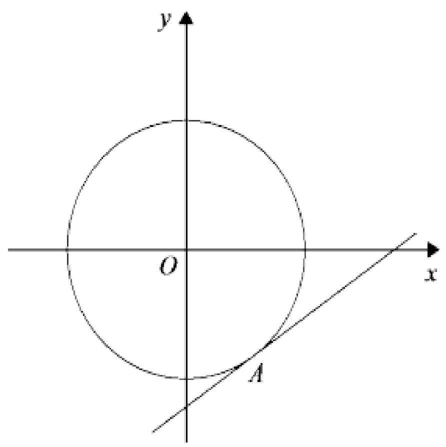
- Recognise a linear, quadratic, cubic, reciprocal and circle graph from its shape;
- Generate points and plot graphs of simple quadratic functions, then more general quadratic functions;
- Find approximate solutions of a quadratic equation from the graph of the corresponding quadratic function;
- Interpret graphs of quadratic functions from real-life problems;
- Draw graphs of simple cubic functions using tables of values;
- Interpret graphs of simple cubic functions, including finding solutions to cubic equations;
- Draw graphs of the reciprocal function with $x \neq 0$ using tables of values;
- Draw circles, centre the origin, equation $x^2 + y^2 = r^2$.

POSSIBLE SUCCESS CRITERIA/EXAM QUESTIONS

Select and use the correct mathematical techniques to draw linear, quadratic, cubic and reciprocal graphs.

Identify a variety of functions by the shape of the graph.

The diagram shows the circle with equation $x^2 + y^2 = 261$



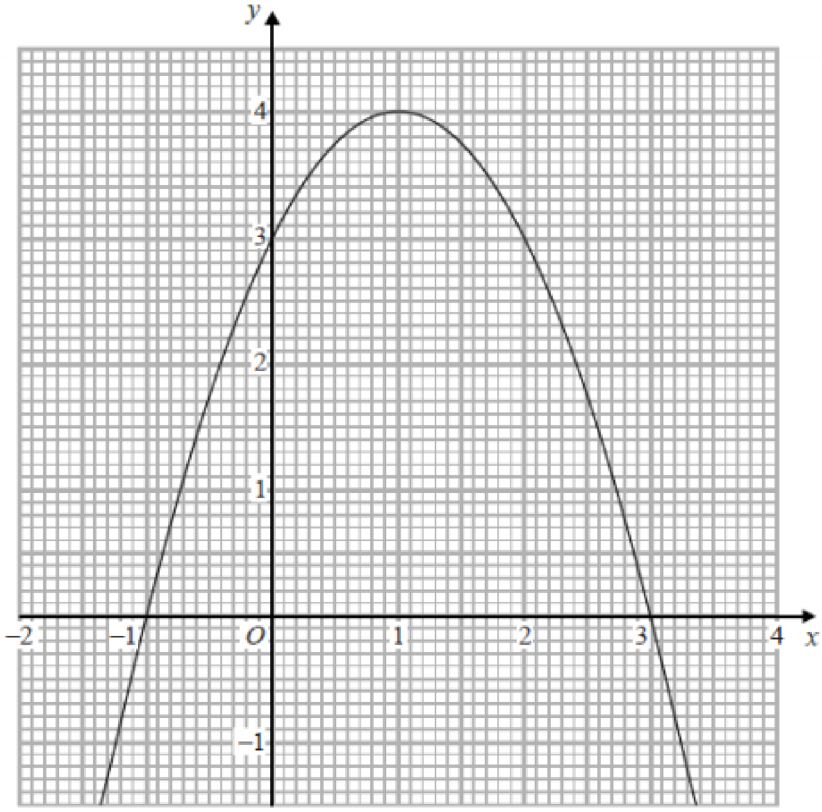
A tangent to the circle is drawn at point A with coordinates $(p, -15)$, where $p > 0$

Find an equation of the tangent at A.

(Total 5 marks)

Mock Papers Set 3, Paper 3H qu.22 (A16, A9, A10 – AO1/AO3)

The graph of $y = f(x)$ is drawn on the grid.



(*a*) Write down the coordinates of the turning point of the graph.

(1)

(*b*) Write down the roots of $f(x) = 2$

(1)

(*c*) Write down the value of $f(0.5)$

(1)

(Total 3 marks)

New SAMs Paper 2H qu.7 (A11, A7 – AO2)

OPPORTUNITIES FOR REASONING/PROBLEM SOLVING

Match equations of quadratics and cubics with their graphs by recognising the shape or by sketching.

COMMON MISCONCEPTIONS

Students struggle with the concept of solutions and what they represent in concrete terms.

NOTES

Use lots of practical examples to help model the quadratic function, e.g. draw a graph to model the trajectory of a projectile and predict when/where it will land.

Ensure axes are labelled and pencils used for drawing.
Graphical calculations or appropriate ICT will allow students to see the impact of changing variables within a function.

3c. Scatter graphs (S4, S6)	Teaching time 4–6 hours

OBJECTIVES

By the end of the sub-unit, students should be able to:

- Draw and interpret scatter graphs;
- Interpret scatter graphs in terms of the relationship between two variables;
- Draw lines of best fit by eye, understanding what these represent;
- Identify outliers and ignore them on scatter graphs;
- Use a line of best fit, or otherwise, to predict values of a variable given values of the other variable;
- Distinguish between positive, negative and zero correlation using lines of best fit, and interpret correlation in terms of the problem;
- Understand that correlation does not imply causality, and appreciate that correlation is a measure of the strength of the association between two variables and that zero correlation does not necessarily imply 'no relationship' but merely 'no linear correlation';
- Explain an isolated point on a scatter graph;
- Use the line of best fit make predictions; interpolate and extrapolate apparent trends whilst knowing the dangers of so doing.

POSSIBLE SUCCESS CRITERIA/EXAM QUESTIONS

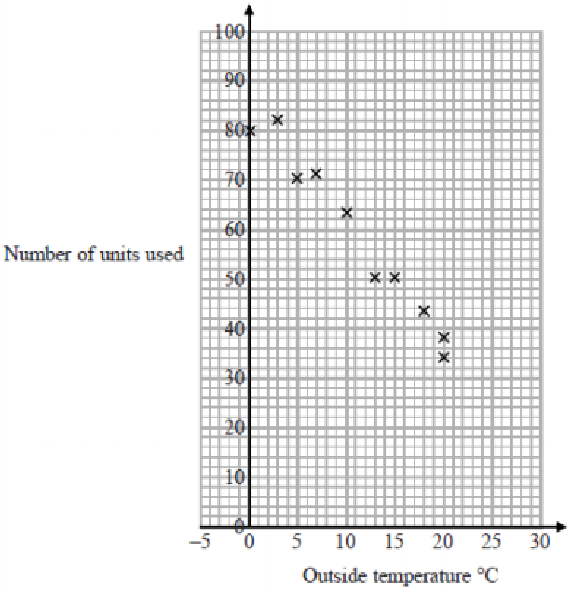
Be able to justify an estimate they have made using a line of best fit.

Identify outliers and explain why they may occur.

Given two sets of data in a table, model the relationship and make predictions.

In a survey, the outside temperature and the number of units of electricity used for heating were recorded for ten homes.

The scatter diagram shows this information.



Molly says,

"On average the number of units of electricity used for heating decreases by 4 units for each °C increase in outside temperature."

(*a*) Is Molly right?
Show how you get your answer.

(3)

(*b*) You should **not** use a line of best fit to predict the number of units of electricity used for heating when the outside temperature is 30°C.

Give one reason why.

(1)

(Total 4 marks)

New SAMs Paper 3F qu.21 / 3H qu.4 (S6, A10 – AO2)

OPPORTUNITIES FOR REASONING/PROBLEM SOLVING

Many real-life situations that give rise to two variables provide opportunities for students to extrapolate and interpret the resulting relationship (if any) between the variables.

COMMON MISCONCEPTIONS

Students often forget the difference between continuous and discrete data.

Lines of best fit are often forgotten, but correct answers still obtained by sight.

NOTES

Students need to be constantly reminded of the importance of drawing a line of best fit.

A possible extension includes drawing the line of best fit through the mean point (mean of x, mean of y).