7b. 3D forms and volume, cylinders, cones and spheres (N8, N14, N15, A5, A21, G1, G12, G13, G14, G16, G17)	**Teaching time** 7–9 hours

OBJECTIVES

By the end of the sub-unit, students should be able to:

- Find the surface area of prisms using the formulae for triangles and rectangles, and other (simple) shapes with and without a diagram;
- Draw sketches of 3D solids;
- Identify planes of symmetry of 3D solids, and sketch planes of symmetry;
- Recall and use the formula for the volume of a cuboid or prism made from composite 3D solids using a variety of metric measures;
- Convert between metric volume measures;
- Convert between metric measures of volume and capacity, e.g. 1 ml = 1 cm^3;
- Use volume to solve problems;
- Estimating surface area, perimeter and volume by rounding measurements to 1 significant figure to check reasonableness of answers.
- Use $\pi \approx 3.142$ or use the π button on a calculator;
- Find the volume and surface area of a cylinder;
- Recall and use the formula for volume of pyramid;
- Find the surface area of a pyramid;
- Use the formulae for volume and surface area of spheres and cones;
- Solve problems involving more complex shapes and solids, including segments of circles and frustums of cones;
- Find the surface area and volumes of compound solids constructed from cubes, cuboids, cones, pyramids, spheres, hemispheres, cylinders;
- Give answers to an appropriate degree of accuracy or in terms of π;
- Form equations involving more complex shapes and solve these equations.

POSSIBLE SUCCESS CRITERIA/EXAM QUESTIONS

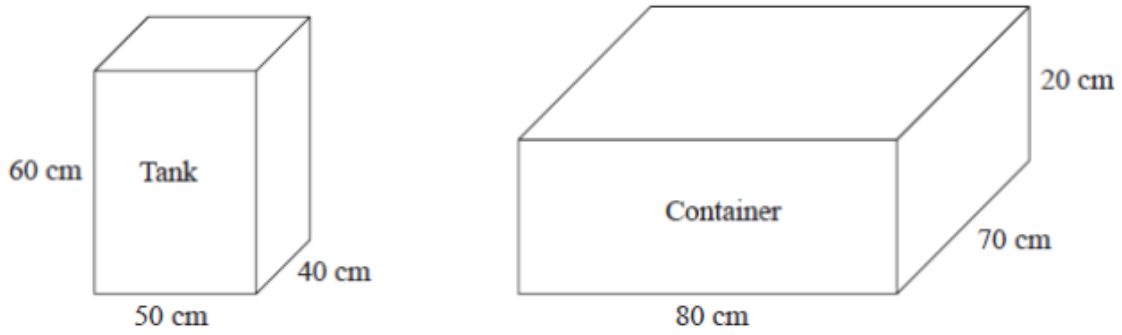
Given dimensions of a rectangle and a pictorial representation of it when folded, work out the dimensions of the new shape.

Work out the length given the area of the cross-section and volume of a cuboid.

Understand that answers in terms of π are more accurate.

Given two solids with the same volume and the dimensions of one, write and solve an equation in terms of π to find the dimensions of the other, e.g. a sphere is melted down to make ball bearings of a given radius, how many will it make?

The diagram shows a tank in the shape of a cuboid.
It also shows a container in the shape of a cuboid.



The tank is full of oil.
The container is empty

35% of the oil from the tank is spilled.
The rest of the oil from the tank is put into the container.

Work out the height of the oil in the container.
Give your answer to an appropriate degree of accuracy.

(Total 5 marks)

New SAMs Paper 2F qu.14 (G16, R9 – AO1/AO3)

OPPORTUNITIES FOR REASONING/PROBLEM SOLVING

Combinations of 3D forms such as a cone and a sphere where the radius has to be calculated given the total height.

COMMON MISCONCEPTIONS

Students often get the concepts of surface area and volume confused.

NOTES

Encourage students to draw a sketch where one isn't provided.

Use lots of practical examples to ensure that students can distinguish between surface area and volume. Making solids using multi-link cubes can be useful.

Solve problems including examples of solids in everyday use.

Drawing 3D shapes in 2D using isometric grids isn't an explicit objective but provides an ideal introduction to the topic and for some students provides the scaffolding needed when drawing 3D solids.

Scaffold drawing 3D shapes by initially using isometric paper.

Whilst not an explicit objective, it is useful for students to draw and construct nets and show how they fold to make 3D solids, allowing students to make the link between 3D shapes and their nets. This will enable students to understand that there is often more than one net that can form a 3D shape.

Formulae for curved surface area and volume of a sphere, and surface area and volume of a cone will be given on the formulae page of the examinations.

Ensure that students know it is more accurate to leave answers in terms of π but only when asked to do so.

7c. Accuracy and bounds (N15, N16)	**Teaching time** 5–7 hours

OBJECTIVES

By the end of the sub-unit, students should be able to:

- Calculate the upper and lowers bounds of numbers given to varying degrees of accuracy;
- Calculate the upper and lower bounds of an expression involving the four operations;
- Find the upper and lower bounds in real-life situations using measurements given to appropriate degrees of accuracy;
- Find the upper and lower bounds of calculations involving perimeters, areas and volumes of 2D and 3D shapes;
- Calculate the upper and lower bounds of calculations, particularly when working with measurements;
- Use inequality notation to specify an error interval due to truncation or rounding.

POSSIBLE SUCCESS CRITERIA/EXAM QUESTIONS

Round 16,000 people to the nearest 1000.

Round 1100 g to 1 significant figure.

Work out the upper and lower bounds of a formula where all terms are given to 1 decimal place.

Be able to justify that measurements to the nearest whole unit may be inaccurate by up to one half in either direction.

$m = \frac{\sqrt{s}}{t}$

$s = 3.47$ correct to 3 significant figures

$t = 8.132$ correct to 4 significant figures

By considering bounds, work out the value of m to a suitable degree of accuracy.
Give a reason for your answer.

(Total 5 marks)

New SAMs Paper 2H qu.18 (N16 – AO1/AO2)

OPPORTUNITIES FOR REASONING/PROBLEM SOLVING

This sub-unit provides many opportunities for students to evaluate their answers and provide counter-arguments in mathematical and real-life contexts, in addition to requiring them to understand the implications of rounding their answers.

COMMON MISCONCEPTIONS

Students readily accept the rounding for lower bounds, but take some convincing in relation to upper bounds.

NOTES

Students should use ‘half a unit above’ and ‘half a unit below’ to find upper and lower bounds.

Encourage use a number line when introducing the concept.

UNIT 11: Multiplicative reasoning: direct and inverse proportion, relating to graph form for direct, compound measures, repeated proportional change	**Teaching Time** 7–9 hours

SPECIFICATION REFERENCES

N3 recognise and use relationships between operations, including inverse operations (e.g. cancellation to simplify calculations and expressions); …
N12 interpret fractions and percentages as operators
N13 use standard units of mass, length, time, money and other measures (including standard compound measures) using decimal quantities where appropriate
R1 change freely between related standard units (e.g. time, length, area, volume/capacity, mass) and compound units (e.g. speed, rates of pay, prices, density, pressure) in numerical and algebraic contexts
R6 express a multiplicative relationship between two quantities as a ratio or a fraction
R8 relate ratios to fractions and to linear functions
R10 solve problems involving direct and inverse proportion, including graphical and algebraic representations
R11 use compound units such as speed, rates of pay, unit pricing, density and pressure
R13 understand that X is inversely proportional to Y is equivalent to X is proportional to ; …
R14 … recognise and interpret graphs that illustrate direct and inverse proportion
R16 set up, solve and interpret the answers in growth and decay problems, including compound interest **and work with general iterative processes**

PRIOR KNOWLEDGE

Students should be able to find a percentage of an amount and relate percentages to decimals.

Students should be able to rearrange equations and use these to solve problems.

Knowledge of speed = distance/time, density = mass/volume.

KEYWORDS

Tier 2
Value, mass, volume, speed, distance, time, volume, pressure, direct

Tier 3
Ratio, proportion, best value, unitary, proportional change, compound measure, density, density, acceleration, velocity, inverse, constant of proportionality

OBJECTIVES

By the end of the unit, students should be able to:

- Express a multiplicative relationship between two quantities as a ratio or a fraction, e.g. when A:B are in the ratio 3:5, A is B. When $4a = 7b$, then $a =$ or a:b is 7:4;
- Solve proportion problems using the unitary method;
- Work out which product offers best value and consider rates of pay;
- Work out the multiplier for repeated proportional change as a single decimal number;
- Represent repeated proportional change using a multiplier raised to a power, use this to solve problems involving compound interest and depreciation;
- Understand and use compound measures and:
  - convert between metric speed measures;
  - convert between density measures;
  - convert between pressure measures;
- Use kinematics formulae from the formulae sheet to calculate speed, acceleration, etc (with variables defined in the question);
- Calculate an unknown quantity from quantities that vary in direct or inverse proportion;
- Recognise when values are in direct proportion by reference to the graph form, and use a graph to find the value of k in $y = kx$;
- Set up and use equations to solve word and other problems involving direct proportion (this is covered in more detail in unit 19);
- Relate algebraic solutions to graphical representation of the equations;
- Recognise when values are in inverse proportion by reference to the graph form;
- Set up and use equations to solve word and other problems involving inverse proportion, and relate algebraic solutions to graphical representation of the equations.

POSSIBLE SUCCESS CRITERIA/EXAM QUESTIONS

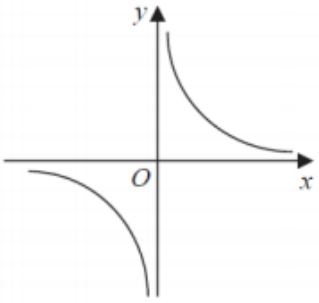
Change g/cm^3 to kg/m^3, kg/m^2 to g/cm^2, m/s to km/h.

Solve word problems involving direct and inverse proportion.

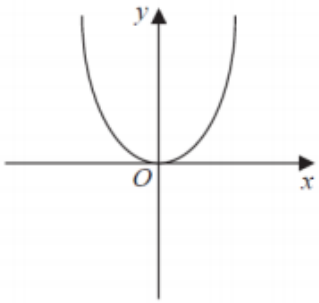
Understand direct proportion as: as x increases, y increases.

Understand inverse proportion as: as x increases, y decreases.

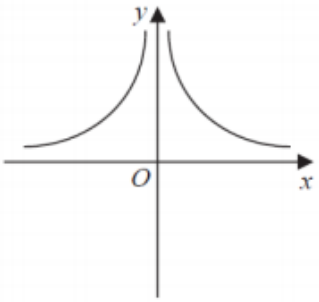
These graphs show four different proportionality relationships between y and x.



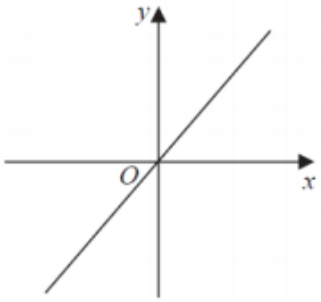
Graph A



Graph B



Graph C



Graph D

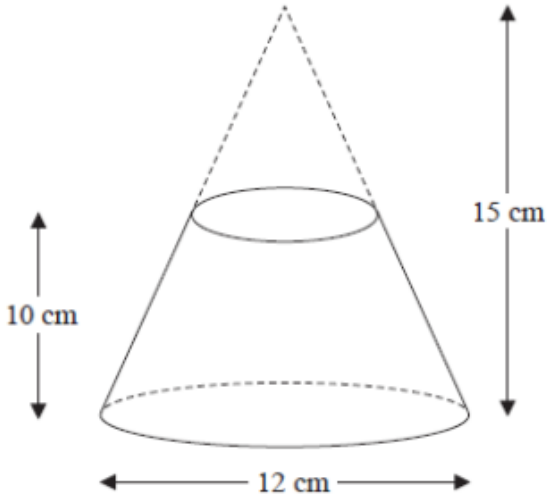
Match each graph with a statement in the table below.

Proportionality relationship	**Graph letter**
y is directly proportional to x	
y is inversely proportional to x	
y is proportional to the square of x	
y is inversely proportional to the square of x	

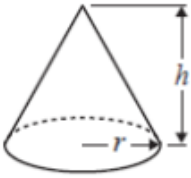
(Total 2 marks)

Specimen Papers Set 1, Paper 1H qu.16 (R14 – AO2)

A frustum is made by removing a small cone from a large cone as shown in the diagram.



Volume of cone $= \frac{1}{3}\pi r^2 h$



The frustum is made from glass.
The glass has a density of 2.5 g/cm^3

Work out the mass of the frustum.
Give your answer to an appropriate degree of accuracy.

(Total 5 marks)

New SAMs Paper 2H qu.22 (R11, N15, G17, G19 – AO1/AO3)

Louis and Robert are investigating the growth in the population of a type of bacteria.
They have two flasks A and B.

At the start of day 1, there are 1000 bacteria in flask A.
The population of bacteria grows exponentially at the rate of 50% per day.

(*a*) Show that the population of bacteria in flask A at the start of each day forms a geometric progression.

(2)

The population of bacteria in flask A at the start of the 10th day is k times the population of bacteria in flask A at the start of the 6th day.

(*b*) Find the value of k.

(2)

At the start of day 1 there are 1000 bacteria in flask B.
The population of bacteria in flask B grows exponentially at the rate of 30% per day.

(*c*) Sketch a graph to compare the size of the population of bacteria in flask A and in flask B.

(1)

(Total 5 marks)

New SAMs Paper 3H qu.17 (R16, A12 – AO1/AO2/AO3)

At the start of year n, the quantity of a radioactive metal is P_n
At the start of the following year, the quantity of the same metal is given by

$$P_{n+1} = 0.87P_n$$

At the start of 2016 there were 30 grams of the metal.

What will be the quantity of the metal at the start of 2019?
Give your answer to the nearest gram.

(Total 3 marks)

Mock Papers Set 3, Paper 3H qu.17 (R16 – AO1)

OPPORTUNITIES FOR REASONING/PROBLEM SOLVING

Speed/distance type problems that involve students justifying their reasons why one vehicle is faster than another.

Calculations involving value for money are a good reasoning opportunity that utilise different skills.

Working out best value of items using different currencies given an exchange rate.

NOTES

Include fractional percentages of amounts with compound interest and encourage use of single multipliers.

Amounts of money should be rounded to the nearest penny, but emphasise the importance of not rounding until the end of the calculation if doing in stages.

Use a formula triangle to help students see the relationship for compound measures – this will help them evaluate which inverse operations to use.

Help students to recognise the problem they are trying to solve by the unit measurement given, e.g. km/h is a unit of speed as it is speed divided by a time.

Kinematics formulae involve a constant acceleration (which could be zero).

Encourage students to write down the initial equation of proportionality and, if asked to find a formal relating two quantities, the constant of proportionality must be found.