5b. Pythagoras' Theorem and trigonometry (N7, N8, N15, A4, A5, R12, G6, G20, G21)	**Teaching time** 7–9 hours

OBJECTIVES

By the end of the sub-unit, students should be able to:

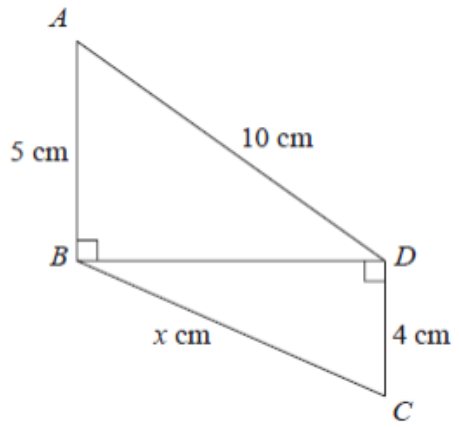
- Understand, recall and use Pythagoras' Theorem in 2D;
- Given three sides of a triangle, justify if it is right-angled or not;
- Calculate the length of the hypotenuse in a right-angled triangle (including decimal lengths and a range of units);
- Find the length of a shorter side in a right-angled triangle;
- Calculate the length of a line segment AB given pairs of points;
- Give an answer to the use of Pythagoras' Theorem in surd form;
- Understand, use and recall the trigonometric ratios sine, cosine and tan, and apply them to find angles and lengths in general triangles in 2D figures;
- Use the trigonometric ratios to solve 2D problems;
- Find angles of elevation and depression;
- Know the exact values of sin θ and cos θ for θ = 0°, 30°, 45°, 60° and 90°; know the exact value of tan θ for θ = 0°, 30°, 45° and 60°.

POSSIBLE SUCCESS CRITERIA/EXAM QUESTIONS

Does 2, 3, 6 give a right-angled triangle?

Justify when to use Pythagoras' Theorem and when to use trigonometry.

Triangles *ABD* and *BCD* are right-angled triangles.

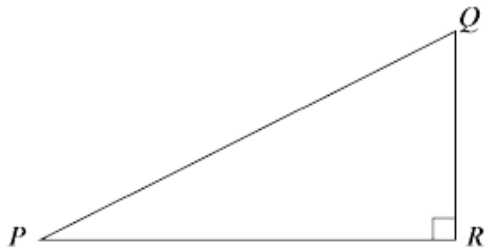


Work out the value of x.
Give your answer correct to 2 decimal places.

(Total 4 marks)

New SAMs Paper 2F qu.28 / 2H qu.6 (G20 – AO1/AO3)

Here is triangle PQR.

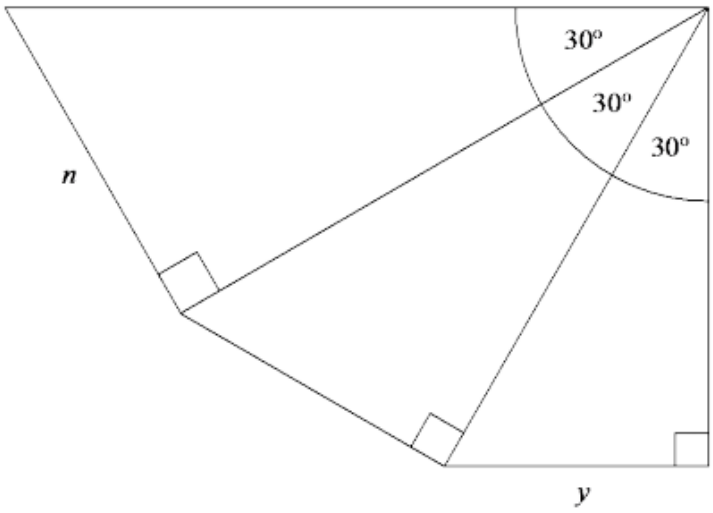


The length of QR is 60% of the length of PR.

Find the value of sin QPR.
Give your answer correct to 3 significant figures.

(Total 3 marks)

Mock Papers Set 3, Paper 2H qu.12 (R6, G20 – AO1/AO3)



The diagram shows three right-angled triangles.

Prove that $y = \frac{3}{4}n$

(Total 4 marks)

Mock Papers Set 3, Paper 1H qu.20 (G21, G21 – AO1/AO2)

OPPORTUNITIES FOR REASONING/PROBLEM SOLVING

Combined triangle problems that involve consecutive application of Pythagoras' Theorem or a combination of Pythagoras' Theorem and the trigonometric ratios.

In addition to abstract problems, students should be encouraged to apply Pythagoras' Theorem and/or the trigonometric ratios to real-life scenarios that require them to evaluate whether their answer fulfils certain criteria, e.g. the angle of elevation of 6.5 m ladder cannot exceed 65°. What is the greatest height it can reach? Rounding skills will be important here when justifying their findings.

COMMON MISCONCEPTIONS

Answers may be displayed on a calculator in surd form.

Students forget to square root their final answer, or round their answer prematurely.

NOTES

Students may need reminding about surds.

Drawing the squares on the three sides will help when deriving the rule.

Scale drawings are not acceptable.

Calculators need to be in degree mode.

To find in right-angled triangles the exact values of sin θ and cos θ for θ = 0°, 30°, 45°, 60° and 90°, use triangles with angles of 30°, 45° and 60°.

Use a suitable mnemonic to remember SOHCAHTOA.

Use Pythagoras' Theorem and trigonometry together.

UNIT 8: Transformations; Constructions: triangles, nets, plan and elevation, loci, scale drawings and bearings

SPECIFICATION REFERENCES

R2 use scale factors, scale diagrams and maps
R6 express a multiplicative relationship between two quantities as a ratio or a fraction
G2 use the standard ruler and compass constructions (perpendicular bisector of a line segment, constructing a perpendicular to a given line from/at a given point, bisecting a given angle); use these to construct given figures and solve loci problems; know that the perpendicular distance from a point to a line is the shortest distance to the line
G3 apply the properties of angles at a point, angles at a point on a straight line, vertically opposite angles; understand and use alternate and corresponding angles on parallel lines; derive and use the sum of angles in a triangle (e.g. to deduce and use the angle sum in any polygon, and to derive properties of regular polygons)
G5 use the basic congruence criteria for triangles (SSS, SAS, ASA, RHS)
G7 identify, describe and construct congruent and similar shapes, including on a coordinate axis, by considering rotation, reflection, translation and enlargement (including fractional **and negative** scale factors)
G8 **describe the changes and invariance achieved by combinations of rotations, reflections and translations**
G12 identify properties of the faces, surfaces, edges and vertices of: cubes, cuboids, prisms, cylinders, pyramids, cones and spheres
G13 construct and interpret plans and elevations of 3D shapes
G15 measure line segments and angles in geometric figures, including interpreting maps and scale drawings and use of bearings
G24 describe translations as 2D vectors
G25 apply addition and subtraction of vectors, multiplication of vectors by a scalar, and diagrammatic and column representations of vectors; …

PRIOR KNOWLEDGE

Students should be able to recognise 2D shapes.

Students should be able to plot coordinates in four quadrants and linear equations parallel to the coordinate axes.

KEYWORDS

Tier 2Translation, transformation, centre, direction, describe, similar, combination, construction

Tier 3 Rotation, reflection, enlargement, scale factor, vector, angle, mirror line, centre of enlargement, congruence, compasses, protractor, bisector, bisect, line segment, perpendicular, loci, bearing

8a. Transformations (R6, G5, G7, G8, G24, G25)	**Teaching time** 7–9 hours

OBJECTIVES

By the end of the sub-unit, students should be able to:

- Distinguish properties that are preserved under particular transformations;
- Recognise and describe rotations – know that that they are specified by a centre and an angle;
- Rotate 2D shapes using the origin or any other point (not necessarily on a coordinate grid);
- Identify the equation of a line of symmetry;
- Recognise and describe reflections on a coordinate grid – know to include the mirror line as a simple algebraic equation, $x = a$, $y = a$, $y = x$, $y = -x$ and lines not parallel to the axes;
- Reflect 2D shapes using specified mirror lines including lines parallel to the axes and also

$y = x$ and $y = -x$;

- Recognise and describe single translations using column vectors on a coordinate grid;
- Translate a given shape by a vector;
- Understand the effect of one translation followed by another, in terms of column vectors (to introduce vectors in a concrete way);
- Enlarge a shape on a grid without a centre specified;
- Describe and transform 2D shapes using enlargements by a positive integer, positive fractional, and negative scale factor;
- Know that an enlargement on a grid is specified by a centre and a scale factor;
- Identify the scale factor of an enlargement of a shape;
- Enlarge a given shape using a given centre as the centre of enlargement by counting distances from centre, and find the centre of enlargement by drawing;

- Find areas after enlargement and compare with before enlargement, to deduce multiplicative relationship (area scale factor); given the areas of two shapes, one an enlargement of the other, find the scale factor of the enlargement (whole number values only);

- Use congruence to show that translations, rotations and reflections preserve length and angle, so that any figure is congruent to its image under any of these transformations;

- Describe and transform 2D shapes using combined rotations, reflections, translations, or enlargements;

- Describe the changes and invariance achieved by combinations of rotations, reflections and translations.

POSSIBLE SUCCESS CRITERIA/EXAM QUESTIONS

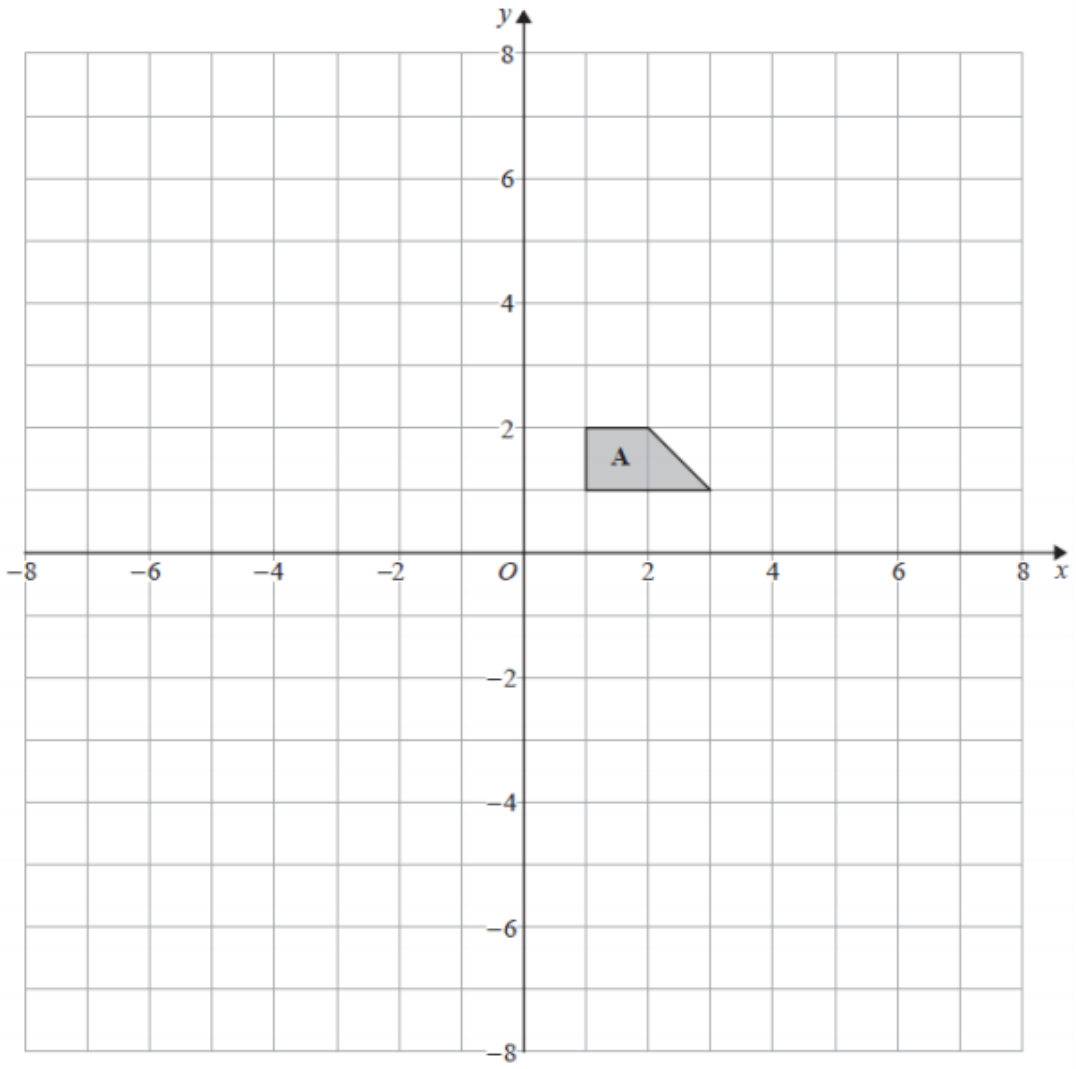
Recognise similar shapes because they have equal corresponding angles and/or sides scaled up in same ratio.

Understand that translations are specified by a distance and direction (using a vector).

Recognise that enlargements preserve angle but not length.

Understand that distances and angles are preserved under rotations, reflections and translations so that any shape is congruent to its image.

Understand that similar shapes are enlargements of each other and angles are preserved.



(*a*) Enlarge shape **A** by scale factor −2, centre (0, 0)
Label your image **B.**

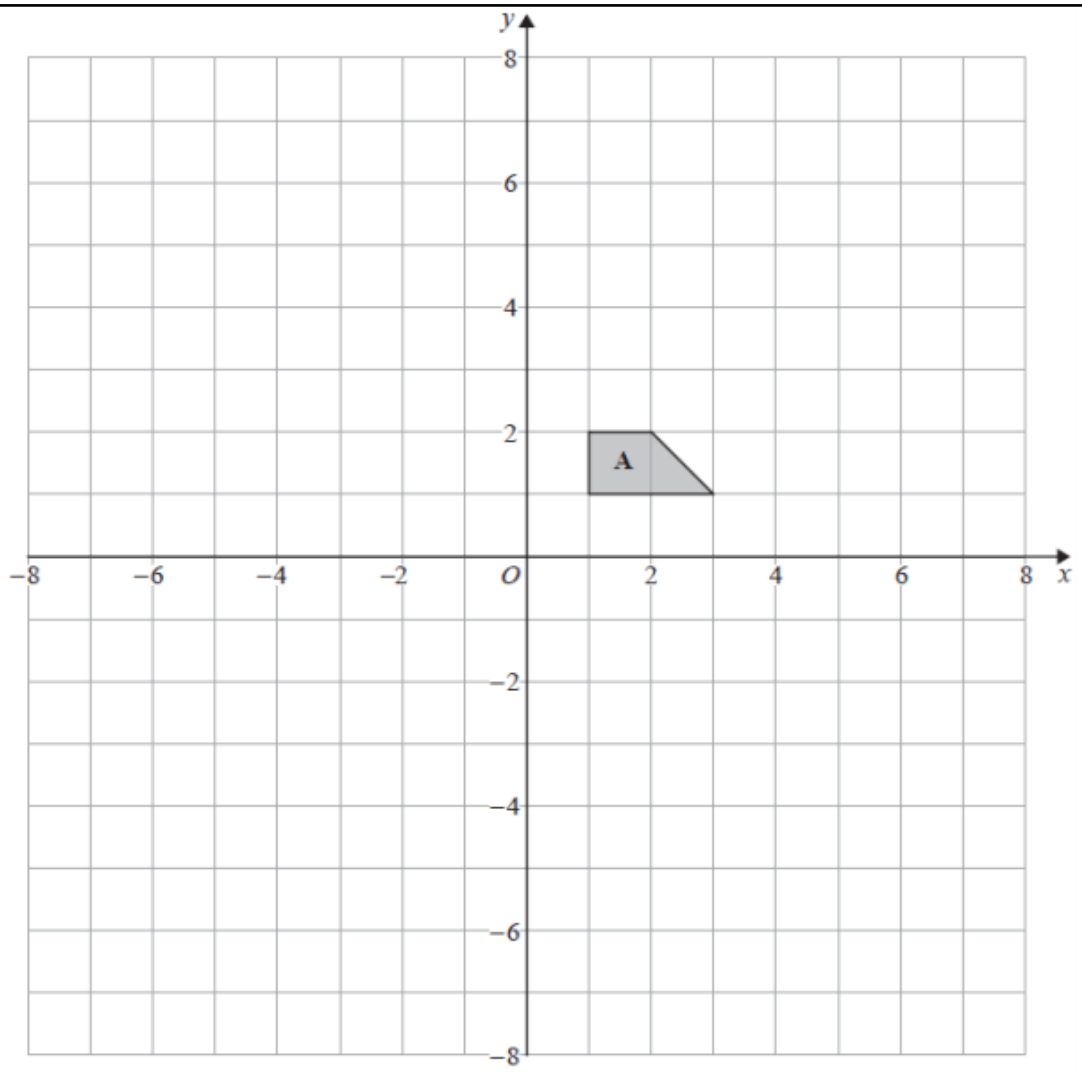
(2)

(*b*) Describe fully the single transformation that will map shape **B** onto shape **A**.

(1)

(Total 3 marks)

New SAMs Paper 1H qu.20 (G7, G8 – AO1/AO2)



(*a*) Enlarge shape **A** by scale factor −2, centre (0, 0)
Label your image **B.**

(2)

(*b*) Describe fully the single transformation that will map shape **B** onto shape **A**.

(1)

(Total 3 marks)

New SAMs Paper 1H qu.20 (G8, G7 – AO1/AO2)

OPPORTUNITIES FOR REASONING/PROBLEM SOLVING

Students should be given the opportunity to explore the effect of reflecting in two parallel mirror lines and combining transformations.

COMMON MISCONCEPTIONS

Students often use the term 'transformation' when describing transformations instead of the required information.

Lines parallel to the coordinate axes often get confused.

NOTES

Emphasise the need to describe the transformations fully, and if asked to describe a 'single' transformation students should not include two types.

Find the centre of rotation, by trial and error and by using tracing paper. Include centres on or inside shapes.

Area of similar shapes is covered in unit 12.

8b. Constructions, loci and bearings (R2, G2, G3, G12, G13, G15)	**Teaching time** 7–9 hours

OBJECTIVES

By the end of the sub-unit, students should be able to:

- Understand and draw front and side elevations and plans of shapes made from simple solids;
- Given the front and side elevations and the plan of a solid, draw a sketch of the 3D solid;
- Use and interpret maps and scale drawings, using a variety of scales and units;
- Read and construct scale drawings, drawing lines and shapes to scale;
- Estimate lengths using a scale diagram;
- Understand, draw and measure bearings;
- Calculate bearings and solve bearings problems, including on scaled maps, and find/mark and measure bearings
- Use the standard ruler and compass constructions:
  - bisect a given angle;
  - construct a perpendicular to a given line from/at a given point;
  - construct angles of 90°, 45°;
  - perpendicular bisector of a line segment;
- Construct:
  - a region bounded by a circle and an intersecting line;
  - a given distance from a point and a given distance from a line;
  - equal distances from two points or two line segments;
  - regions which may be defined by 'nearer to' or 'greater than';
- Find and describe regions satisfying a combination of loci, including in 3D;
- Use constructions to solve loci problems including with bearings;
- Know that the perpendicular distance from a point to a line is the shortest distance to the line.

POSSIBLE SUCCESS CRITERIA/EXAM QUESTIONS

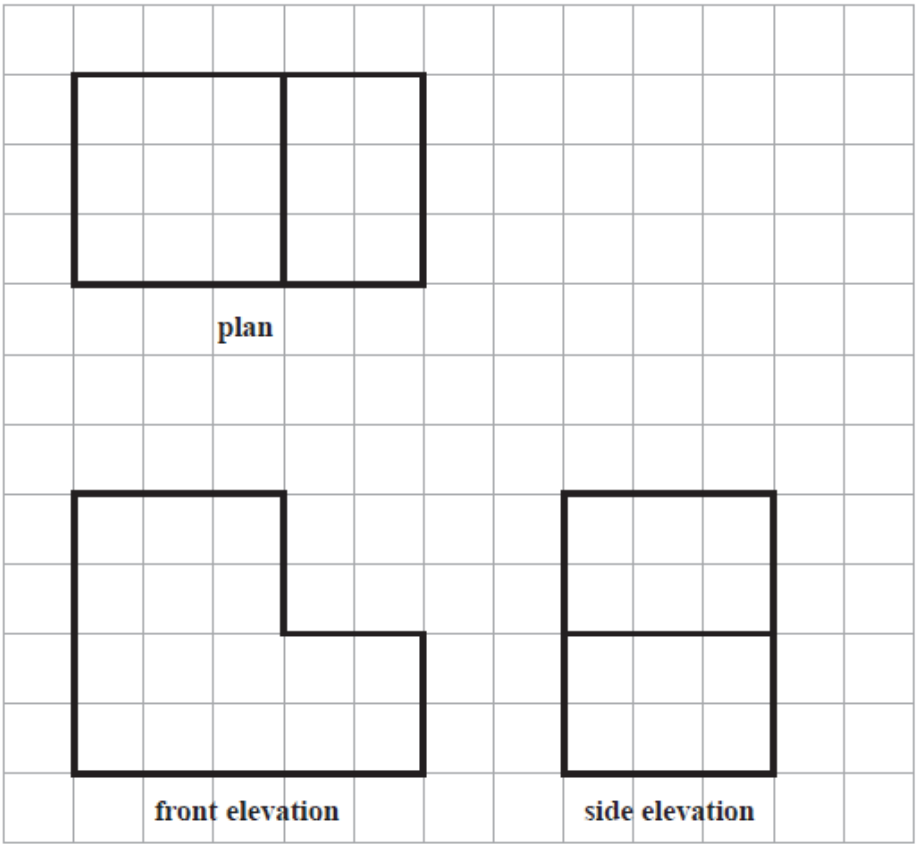
Able to read and construct scale drawings.

When given the bearing of a point A from point B, can work out the bearing of B from A.

Know that scale diagrams, including bearings and maps, are 'similar' to the real-life examples.

Able to sketch the locus of point on a vertex of a rotating shape as it moves along a line, of a point on the circumference and at the centre of a wheel.

The plan, front elevation and side elevation of a solid prism are drawn on a centimetre grid.



In the space below, draw a sketch of the solid prism.
Write the dimensions of the prism on your sketch.

(Total 2 marks)

New SAMs Paper 1F qu.26 / 1H qu.6 (G13 – AO2)

The diagram shows a rectangle *ABCD*.

In the space below, use a ruler and a pair of compasses to construct a right-angled triangle equal in area to the area of the rectangle *ABCD*.

You must show all your construction lines.

The base of the triangle, which is equal in length to the side *CD*, has been drawn for you.

(Total 3 marks)

Original SAMs Paper 3F qu.17 / 3H qu.6 (G2, G16 – AO2)

OPPORTUNITIES FOR REASONING/PROBLEM SOLVING

Interpret a given plan and side view of a 3D form to be able to produce a sketch of the form.

Problems involving combinations of bearings and loci can provide a rich opportunity to link with other areas of mathematics and allow students to justify their findings.

COMMON MISCONCEPTIONS

Correct use of a protractor may be an issue.

NOTES

Drawings should be done in pencil.

Relate loci problems to real-life scenarios, including mobile phone masts and coverage.

Construction lines should not be erased.