

<b>UNIT 12: Similarity and congruence in 2D and 3D</b>	<b>Teaching Time</b>
	<b>7–9 hours</b>

## SPECIFICATION REFERENCES

R6 express a multiplicative relationship between two quantities as a ratio or a fraction

R12 compare lengths, areas and volumes using ratio notation; make links to similarity (including trigonometric ratios) and scale factors

G5 use the basic congruence criteria for triangles (SSS, SAS, ASA, RHS)

G6 apply angle facts, triangle congruence, similarity and properties of quadrilaterals to conjecture and derive results about angles and sides, including ... the fact that the base angles of an isosceles triangle are equal, and use known results to obtain simple proofs

G17 ... calculate: surface area and volume of spheres, pyramids, cones and composite solids

G19 apply the concepts of congruence and similarity, including the relationships between lengths, areas and volumes in similar figures

## PRIOR KNOWLEDGE

Students should be able to recognise and enlarge shapes and calculate scale factors.

Students should have knowledge of how to calculate area and volume in various metric measures.

Students should be able to measure lines and angles, and use compasses, ruler and protractor to construct standard constructions.

## KEYWORDS

### Tier 2

Side, shape, volume, area, length, similar

### Tier 3

Congruence, angle, compass, construction, scale factor, enlargement, perimeter, frustum

## OBJECTIVES

By the end of the unit, students should be able to:

- Understand and use SSS, SAS, ASA and RHS conditions to prove the congruence of triangles using formal arguments, and to verify standard ruler and pair of compasses constructions;
- Solve angle problems by first proving congruence;
- Understand similarity of triangles and of other plane shapes, and use this to make geometric inferences;
- Prove that two shapes are similar by showing that all corresponding angles are equal in size and/or lengths of sides are in the same ratio/one is an enlargement of the other, giving the scale factor;
- Use formal geometric proof for the similarity of two given triangles;
- Understand the effect of enlargement on angles, perimeter, area and volume of shapes and solids;
- Identify the scale factor of an enlargement of a similar shape as the ratio of the lengths of two corresponding sides, using integer or fraction scale factors;
- Write the lengths, areas and volumes of two shapes as ratios in their simplest form;
- Find missing lengths, areas and volumes in similar 3D solids;
- Know the relationships between linear, area and volume scale factors of mathematically similar shapes and solids;
- Use the relationship between enlargement and areas and volumes of simple shapes and solids;
- Solve problems involving frustums of cones where you have to find missing lengths first using similar triangles.

## POSSIBLE SUCCESS CRITERIA/EXAM QUESTIONS

Recognise that all corresponding angles in similar shapes are equal in size when the corresponding lengths of sides are not.

Understand that enlargement does not have the same effect on area and volume.

Understand, from the experience of constructing them, that triangles satisfying SSS, SAS, ASA and RHS are unique, but SSA triangles are not.

Solid A and solid B are mathematically similar.

The ratio of the surface area of solid A to the surface area of solid B is 4 : 9

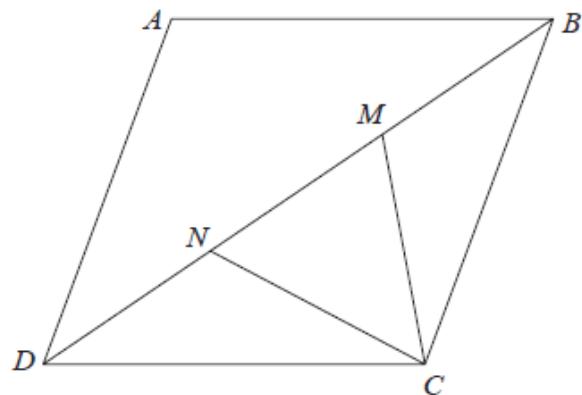
The volume of solid B is  $405 \text{ cm}^3$ .

Show that the volume of solid A is  $120 \text{ cm}^3$ .

(Total 3 marks)

New SAMs Paper 1H qu.18 (G19, R12 – AO2)

$ABCD$  is a rhombus.



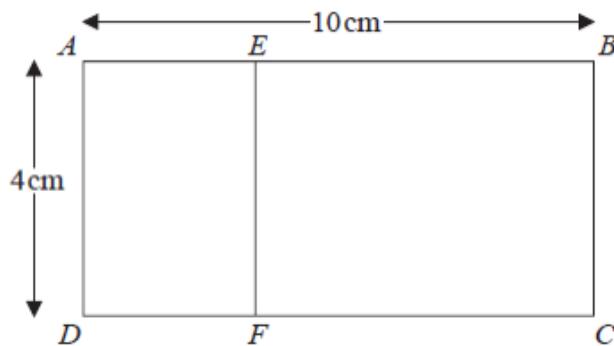
$M$  and  $N$  are points on  $BD$  such that  $DN = MB$ .

Prove that triangle  $DNC$  is congruent to triangle  $BMC$ .

**(Total 3 marks)**

*New SAMS Paper 3H qu.13 (G5, G4 – AO2)*

Rectangle  $ABCD$  is mathematically similar to rectangle  $DAEF$ .



$$AB = 10 \text{ cm.}$$

$$AD = 4 \text{ cm.}$$

Work out the area of rectangle  $DAEF$ .

**(Total 3 marks)**

*New SAMS Paper 1H qu.13 (G6, G13 – AO1/AO3)*

## OPPORTUNITIES FOR REASONING/PROBLEM SOLVING

Multi-step questions which require calculating missing lengths of similar shapes prior to calculating area of the shape, or using this information in trigonometry or Pythagoras problems.

## COMMON MISCONCEPTIONS

Students commonly use the same scale factor for length, area and volume.

## NOTES

Encourage students to model consider what happens to the area when a 1 cm square is enlarged by a scale factor of 3.

Ensure that examples involving given volumes are used, requiring the cube root being calculated to find the length scale factor.

Make links between similarity and trigonometric ratios.