UNIT 13: Sine and cosine rules, ab sin C, trigonometry and Pythagoras' Theorem in 3D, trigonometric graphs, and accuracy and bounds

SPECIFICATION REFERENCES

N16 apply and interpret limits of accuracy, **including upper and lower bounds**

A5 understand and use standard mathematical formulae; rearrange formulae to change the subject

A8 work with coordinates in all four quadrants

A12 recognise, sketch and interpret graphs of linear functions, quadratic functions, simple cubic functions, the reciprocal function with $x \neq 0$, **exponential, functions $y = k^x$ for positive values of k, and the trigonometric functions (with arguments in degrees) y = sin x, y = cos x and y = tan x for angles of any size**

A13 **sketch translations and reflections of a given function**

G11 solve geometrical problems on coordinate axes

G20 know the formulae for: Pythagoras' Theorem $a^2 + b^2 = c^2$ and the trigonometric ratios, sine, cosine and tan; apply them to find angles and lengths in right-angled triangles **and, where possible, general triangles** in two **and three** dimensional figures

G21 know the exact values of sin θ and cos θ for θ = 0°, 30°, 45° , 60° and 90°; know the exact value of tan θ for θ = 0°, 30°, 45° and 60°

G22 **know and apply the sine rule = = , and cosine rule $a^2 = b^2 + c^2 - 2bc$ cos A, to find unknown lengths and angles**

G23 **know and apply Area = ab sin C to calculate the area, sides or angles of any triangle**

PRIOR KNOWLEDGE

Students should be able to use axes and coordinates to specify points in all four quadrants.

Students should be able to recall and apply Pythagoras' Theorem and trigonometric ratios.

Students should be able to substitute into formulae.

KEYWORDS

Tier 2

Transformations, side, plane

Tier 3

Axes, coordinates, sine, cosine, tan, angle, graph, angle, inverse, square root, 2D, 3D, diagonal, cuboid

13a. Graphs of trigonometric functions	**Teaching time**
(A8, A12, A13, G21)	5–7 hours

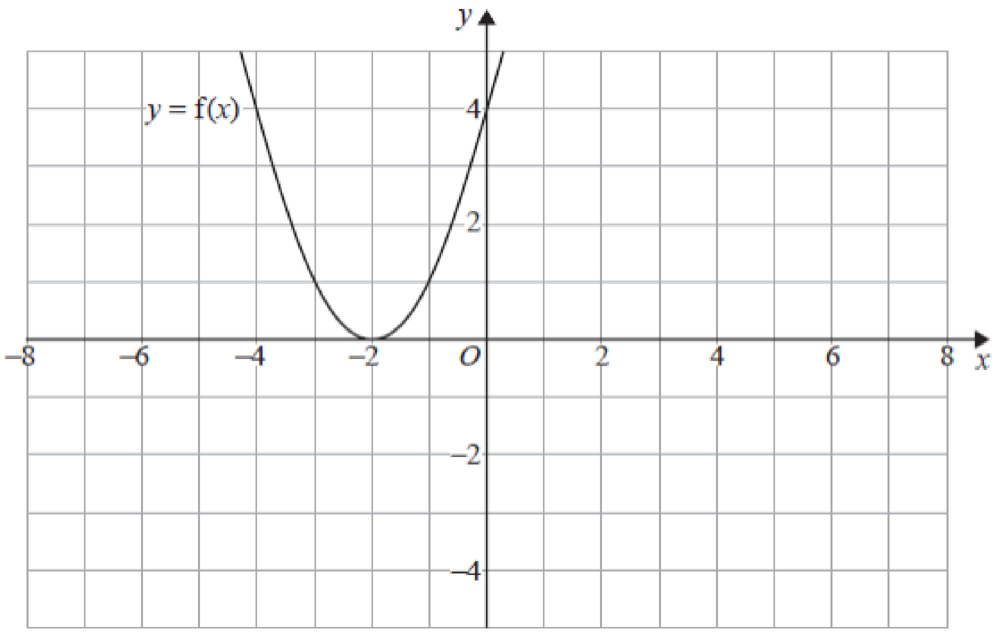
OBJECTIVES

By the end of the sub-unit, students should be able to:

- Recognise, sketch and interpret graphs of the trigonometric functions (in degrees) $y = \sin x$, $y = \cos x$ and $y = \tan x$ for angles of any size.
- Know the exact values of $\sin \theta$ and $\cos \theta$ for $\theta = 0°, 30°, 45°, 60°$ and $90°$ and exact value of $\tan \theta$ for $\theta = 0°, 30°, 45°$ and $60°$ and find them from graphs.
- Apply to the graph of $y = f(x)$ the transformations $y = -f(x)$, $y = f(-x)$ for sine, cosine and tan functions $f(x)$.
- Apply to the graph of $y = f(x)$ the transformations $y = f(x) + a$, $y = f(x + a)$ for sine, cosine and tan functions $f(x)$.

POSSIBLE SUCCESS CRITERIA/EXAM QUESTIONS

Match the characteristic shape of the graphs to their functions and transformations.



(*b*) On this grid, sketch the graph of $y = -f(x) + 3$

(1)

(Total 2 marks)

New SAMs Paper 2H qu.19 (A13, A12 – AO2)

OPPORTUNITIES FOR REASONING/PROBLEM SOLVING

Match a given list of events/processes with their graph.

Calculate and justify specific coordinates on a transformation of a trigonometric function.

NOTES

Translations and reflections of functions are included in this specification, but not rotations or stretches.

This work could be supported by the used of graphical calculators or suitable ICT.

Students need to recall the above exact values for sin, cos and tan.

13b. Further trigonometry	**Teaching time**
(N16, A5, A8, G11, G20, G22, G23)	9–11 hours

OBJECTIVES

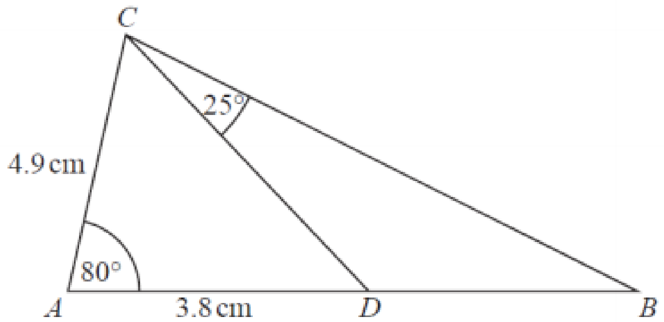
By the end of the sub-unit, students should be able to:

- Know and apply Area = ab sin C to calculate the area, sides or angles of any triangle.
- Know the sine and cosine rules, and use to solve 2D problems (including involving bearings).
- Use the sine and cosine rules to solve 3D problems.
- Understand the language of planes, and recognise the diagonals of a cuboid.
- Solve geometrical problems on coordinate axes.
- Understand, recall and use trigonometric relationships and Pythagoras' Theorem in right-angled triangles, and use these to solve problems in 3D configurations.
- Calculate the length of a diagonal of a cuboid.
- Find the angle between a line and a plane.

POSSIBLE SUCCESS CRITERIA/EXAM QUESTIONS

Find the area of a segment of a circle given the radius and length of the chord.

Justify when to use the cosine rule, sine rule, Pythagoras' Theorem or normal trigonometric ratios to solve problems.



ABC is a triangle.
D is a point on *AB*.

Work out the area of triangle *BCD*.
Give your answer correct to 3 significant figures.

(Total 5 marks)

Specimen Papers Set 2, Paper 3H qu.21 (G22, G23 – AO1/AO3)

In triangle RPQ,

$RP = 8.7$ cm
$PQ = 5.2$ cm
Angle $PRQ = 32°$

(*a*) Assuming that angle PQR is an acute angle,
calculate the area of triangle RPQ.
Give your answer correct to 3 significant figures.

(4)

(*b*) If you did not know that angle PQR is an acute angle, what effect would this have on your calculation of the area of triangle RPQ?

(1)

(Total 5 marks)

New SAMs Paper 2H qu.21 (G23, A12 – AO1/AO3)

OPPORTUNITIES FOR REASONING/PROBLEM SOLVING

Triangles formed in a semi-circle can provide links with other areas of mathematics.

COMMON MISCONCEPTIONS

Not using the correct rule, or attempting to use 'normal trig' in non-right-angled triangles.

When finding angles students will be unable to rearrange the cosine rule or fail to find the inverse of cos θ.

NOTES

The cosine rule is used when we have SAS and used to find the side opposite the 'included' angle or when we have SSS to find an angle.

Ensure that finding angles with 'normal trig' is refreshed prior to this topic.

Students may find it useful to be reminded of simple geometrical facts, i.e. the shortest side is always opposite the shortest angle in a triangle.

The sine and cosine rules and general formula for the area of a triangle are not given on the formulae sheet.

In multi-step questions emphasise the importance of not rounding prematurely and using exact values where appropriate.

Whilst 3D coordinates are not included in the programme of study, they provide a visual introduction to trigonometry in 3D.

UNIT 15: Quadratics, expanding more than two brackets, sketching graphs, graphs of circles, cubes and quadratics	Teaching time 7–9 hours

SPECIFICATION REFERENCES

N8 Calculate exactly with … **surds** …

A4 simplify and manipulate algebraic expressions … by: expanding products of two **or more** binomials

A11 identify and interpret roots, intercepts, turning points of quadratic functions graphically; … **identify turning points by completing the square**

A12 recognise, sketch and interpret graphs of … quadratic functions, simple cubic functions …

A18 solve quadratic equations (**including those that require rearrangement)** …; find approximate solutions using a graph

A19 solve two simultaneous equations in two variables (linear/linear **or linear/quadratic**) algebraically; find approximate solutions using a graph

A20 **find approximate solutions to equations numerically using iteration**

A21 … derive an equation (or two simultaneous equations), solve the equation(s) and interpret the solution.

A22 solve linear inequalities in one or **two** variable**(s), and quadratic inequalities in one variable**; represent the solution set on a number line, **using set notation and on a graph**

R16 … **work with general iterative processes**

PRIOR KNOWLEDGE

Students should be able to solve quadratics and linear equations.

Students should be able to solve simultaneous equations algebraically.

KEYWORDS

Tier 2

Sketch, estimate, function

Tier 3

Quadratic, cubic, factorising, simultaneous equation, graphical, algebraic

OBJECTIVES

By the end of the unit, students should be able to:

- Sketch a graph of a quadratic function, by factorising or by using the formula, identifying roots, y-intercept and turning point by completing the square;
- Be able to identify from a graph if a quadratic equation has any real roots;
- Find approximate solutions to quadratic equations using a graph;
- Expand the product of more than two linear expressions;
- Sketch a graph of a quadratic function and a linear function, identifying intersection points;
- Sketch graphs of simple cubic functions, given as three linear expressions;
- Solve simultaneous equations graphically:
  - find approximate solutions to simultaneous equations formed from one linear function and one quadratic function using a graphical approach;
  - find graphically the intersection points of a given straight line with a circle;
  - solve simultaneous equations representing a real-life situation graphically, and interpret the solution in the context of the problem;
- Solve quadratic inequalities in one variable, by factorising and sketching the graph to find critical values;
- Represent the solution set for inequalities using set notation, i.e. curly brackets and 'is an element of' notation;
  - for problems identifying the solutions to two different inequalities, show this as the intersection of the two solution sets, i.e. solution of $x^2 - 3x - 10 < 0$ as $\{x: -3 < x < 5\}$;
- Solve linear inequalities in two variables graphically;
- Show the solution set of several inequalities in two variables on a graph;
- Use iteration with simple converging sequences.

POSSIBLE SUCCESS CRITERIA/EXAM QUESTIONS

Expand $x(x - 1)(x + 2)$.

Expand $(x - 1)^3$.

Expand $(x + 1)(x + 2)(x - 1)$.

Sketch $y = (x + 1)^2(x - 2)$.

Interpret a pair of simultaneous equations as a pair of straight lines and their solution as the point of intersection.

Be able to state the solution set of $x^2 - 3x - 10 < 0$ as $\{x: x < -3\}$ $\{x: x > 5\}$.

(*a*) Show that the equation $x^3 - 3x^2 + 3 = 0$ has a solution between $x = 2$ and $x = 3$

(2)

(*b*) Show that the equation $x^3 - 3x^2 + 3 = 0$ can be rearranged to give $x = \sqrt[3]{3x^2 - 3}$

(1)

(*c*) Starting with $x_0 = 2$, use the iteration formula $x_{n+1} = \sqrt[3]{3x^2 - 3}$ to find the value of x_2.
Give your answer correct to 3 decimal places.

(3)

(Total 6 marks)
Mock Papers Set 3, Paper 2H qu.16 (A20, A4 AO1/AO2)

Solve $2x^2 - 5x - 12 > 0$

(Total 3 marks)
Mock Papers Set 2, Paper 3H qu.19 (A22 – AO1)

Solve the inequality $x^2 > 3(x + 6)$

(Total 4 marks)
Specimen Papers Set 2, Paper 1H qu.21 (A22 –AO1)

OPPORTUNITIES FOR REASONING/PROBLEM SOLVING

Match equations to their graphs and to real-life scenarios.

"Show that"-type questions will allow students to show a logical and clear chain of reasoning.

COMMON MISCONCEPTIONS

When estimating values from a graph, it is important that students understand it is an 'estimate'.

It is important to stress that when expanding quadratics, the x terms are also collected together.

Quadratics involving negatives sometimes cause numerical errors.

NOTES

The extent of algebraic iteration required needs to be confirmed.

You may want to extend the students to include expansions of more than three linear expressions.

Practise expanding 'double brackets' with all combinations of positives and negatives.
Set notation is a new topic.