UNIT 5: Equations, inequalities and sequences

SPECIFICATION REFERENCES

N1 order positive and negative integers, decimals and fractions; use the symbols =, ≠, <, >, ≤, ≥

N15 round numbers and measures to an appropriate degree of accuracy (e.g. to a specified number of decimal places or significant figures); use inequality notation to specify simple error intervals due to truncation or rounding

N16 apply and interpret limits of accuracy

A2 substitute numerical values into formulae and expressions, including scientific formulae

A3 understand and use the concepts and vocabulary of expressions, equations, formulae, identities, inequalities, terms and factors

A5 understand and use standard mathematical formulae; rearrange formulae to change the subject

A7 where appropriate, interpret simple expressions as functions with inputs and outputs

A17 solve linear equations in one unknown algebraically (including those with the unknown on both sides of the equation); find approximate solutions using a graph

A21 translate simple situations or procedures into algebraic expressions or formulae; derive an equation, solve the equation and interpret the solution

A22 solve linear inequalities in one variable; represent the solution set on a number line

A23 generate terms of a sequence from either a term-to-term or a position-to-term rule

A24 recognise and use sequences of triangular, square and cube numbers, simple arithmetic progressions; Fibonacci type sequences and simple geometric progressions (r^n where n is an integer, and r is a rational number > 0)

A25 deduce expressions to calculate the nth term of linear sequences.

PRIOR KNOWLEDGE

Students should be able to use inequality signs between numbers.
Students should be able to use negative numbers with the four operations, recall and use the hierarchy of operations and understand inverse operations.
Students should be able to deal with decimals and negatives on a calculator.
Students should be able to use index laws numerically.
Students should be able to draw a number line.

KEYWORDS

Tier 2
Inequality, sequence, term, linear, solve, formula, substitution, subject
Tier 3
Equation, integer, nth term, arithmetic, geometric, quadratic, expand

SMSC/RWCM/CEIAG
Needed or problem solving by generalising. Following patterns. Used in engineering and medical professions as well as understanding sales trends and predictions.

5a. Equations	Teaching time
(N1, A2, A3, A5, A17, A21)	6–8 hours

OBJECTIVES

By the end of the sub-unit, students should be able to:
- Select an expression/equation/formula/identity from a list;
- Write expressions and set up simple equations;
- Use function machines;
- Solve simple equations;
- Solve linear equations, with integer coefficients, in which the unknown appears on either side or on both sides of the equation;
- Solve linear equations which contain brackets, including those that have negative signs occurring anywhere in the equation, and those with a negative solution;
- Solve linear equations in one unknown, with integer or fractional coefficients;
- Rearrange simple equations;
- Substitute into a formula, and solve the resulting equation;
- Find an approximate solution to a linear equation using a graph;
- Solve angle or perimeter problems using algebra.
- Write an equation to solve a word problem.

POSSIBLE SUCCESS CRITERIA/EXAM QUESTIONS

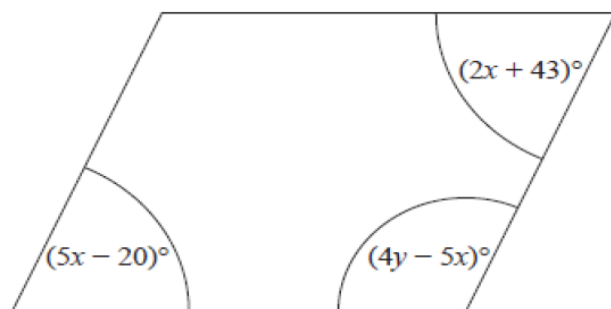Solve: $x + 5 = 12$
Solve: $x - 6 = 3$
Solve: $\dfrac{x}{2} = 5$
Solve: $2x - 5 = 19$
Solve: $2x + 5 = 8x - 7$
Given expressions for the angles on a line or in a triangle in terms of a, find the value of a.
Given expressions for the sides of a rectangle and the perimeter, form and solve an equation to find missing values.

Here is a parallelogram.

Work out the value of x and the value of y.

(Total 5 marks)
New SAMs Paper 1F qu.28 / 1H qu.8 (A21, G4 – AO1/AO3)

(b) Solve $3x - 5 = 9$

(2)

(Total 3 marks)
New SAMs Paper 2F qu.10 (A17, A21 – AO1)

Solve $4x + 5 = x + 26$

(Total 2 marks)
New SAMs Paper 1F qu.19 (A17 – AO1)

OPPORTUNITIES FOR REASONING/PROBLEM SOLVING
Problems that:
* could be solved by forming equations such as: Pat and Paul have a combined salary of £800 per week. Pat earns £200 per week more than Paul. How much does Paul earn?
* involve the application of a formula with conflicting results such as: Pat and Paul are using the formula $y = 8n + 4$ When $n = 2$, Pat states that $y = 86$ and Paul states $y = 20$. Who is correct?

COMMON MISCONCEPTIONS

Rules of adding and subtracting negatives.
Inverse operations can be misapplied.

NOTES

Emphasise good use of notation.
Students need to realise that not all linear equations can be solved by observation or trial and improvement, and hence the use of a formal method is important.
Students can leave their answer in fraction form where appropriate.

5b. Inequalities	Teaching time
(N1, N15, N16, A7, A22)	4–6 hours

OBJECTIVES

By the end of the sub-unit, students should be able to:
* Show inequalities on number lines;
* Write down whole number values that satisfy an inequality;
* Solve an inequality such as $-3 < 2x + 1 < 7$ and show the solution set on a number line;
* Solve two inequalities in x, find the solution sets and compare them to see which value of x satisfies both;
* Use the correct notation to show inclusive and exclusive inequalities;
* Construct inequalities to represent a set shown on a number line;
* Solve simple linear inequalities in one variable, and represent the solution set on a number line;
* Round answers to a given degree of accuracy.
* Use inequality notation to specify simple error intervals due to truncation or rounding.

POSSIBLE SUCCESS CRITERIA/EXAM QUESTIONS

Solve $-3 < 2x + 1$ and show the solution set on a number line.
State the whole numbers that satisfy a given inequality.
Recognise that measurements given to the nearest whole unit may be inaccurate by up to one half in either direction.

(b) Solve $6x + 4 > x + 17$

(2)

(c) n is an integer with $-5 < 2n \leq 6$
Write down all the values of n

(2)

(Total 6 marks)
New SAMs Paper 3H qu.9 (N1, A4, A22 – AO1)

COMMON MISCONCEPTIONS

When solving inequalities, students often state their final answer as a number quantity and either exclude the inequality or change it to =.

NOTES

Emphasise the importance of leaving their answer as an inequality (and not change to =).

5c. Sequences	Teaching time
(A7, A23, A24, A25)	6–8 hours

OBJECTIVES

By the end of the sub-unit, students should be able to:
- Recognise sequences of odd and even numbers, and other sequences including Fibonacci sequences;
- Use function machines to find terms of a sequence;
- Write the term-to-term definition of a sequence in words;
- Find a specific term in the sequence using position-to-term or term-to-term rules;
- Generate arithmetic sequences of numbers, triangular number, square and cube integers and sequences derived from diagrams;
- Recognise such sequences from diagrams and draw the next term in a pattern sequence;
- Find the next term in a sequence, including negative values;
- Find the nth term for a pattern sequence;
- Find the nth term of a linear sequence;
- Find the nth term of an arithmetic sequence;
- Use the nth term of an arithmetic sequence to generate terms;
- Use the nth term of an arithmetic sequence to decide if a given number is a term in the sequence, or find the first term over a certain number;
- Use the nth term of an arithmetic sequence to find the first term greater/less than a certain number;
- Continue a geometric progression and find the term-to-term rule, including negatives, fraction and decimal terms;
- Continue a quadratic sequence and use the nth term to generate terms;
- Distinguish between arithmetic and geometric sequences.

POSSIBLE SUCCESS CRITERIA/EXAM QUESTIONS

Given a sequence, 'Which is the 1st term greater than 50?'
What is the amount of money after x months saving the same amount or the height of tree that grows 6 m per year?
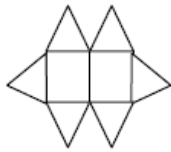What are the next terms in the following sequences?
1, 3, 9, …   100, 50, 25, …   2, 4, 8, 16, …
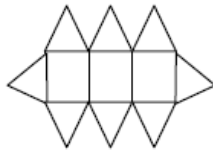Write down an expression for the nth term of the arithmetic sequence 2, 5, 8, 11, …
Is 67 a term in the sequence 4, 7, 10, 13, …?

Here are the first three patterns in a sequence.
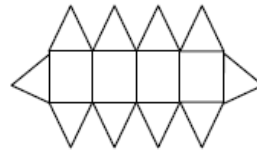The patterns are made from triangles and rectangles.

pattern number 1   pattern number 2   pattern number 3

(a) How many triangles are there in pattern number 7?

(2)

Charlie says

   "There are 4 rectangles in pattern number 3 so there will be 8 rectangles in pattern number 6"

(b) Is Charlie right?
     Give a reason for your answer.

(1)

(Total 3 marks)
New SAMs Paper 1F qu.12 (A23, A24 – AO1/AO2)

Here are the first six terms of a Fibonacci sequence.

$$1 \qquad 1 \qquad 2 \qquad 3 \qquad 5 \qquad 8$$

The rule to continue a Fibonacci sequence is,

the next term in the sequence is the sum of the two previous terms.

(a) Find the 9th term of this sequence.

(1)

The first three terms of a different Fibonacci sequence are

$$a \qquad b \qquad a + b$$

(b) Show that the 6th term of this sequence is $3a + 5b$

(2)

Given that the 3rd term is 7 and the 6th term is 29,

(c) find the value of a and the value of b.

(3)

(Total 6 marks)
New SAMs Paper 3F qu.20 / 3H qu.3 (A24, A19, A21 – AO1/AO2/AO3)

Here are the first four terms of an arithmetic sequence.

$$6 \qquad 10 \qquad 14 \qquad 18$$

(a) Write an expression, in terms of n, for the nth term of this sequence.

(2)

The nth term of a different arithmetic sequence is $3n + 5$

(b) Is 108 a term of this sequence?
Show how you get your answer.

(2)

(Total 4 marks)
New SAMs Paper 2F qu.25 / 2H qu.3 (A25 – AO1/AO2)

OPPORTUNITIES FOR REASONING/PROBLEM SOLVING
Evaluating statements about whether or not specific numbers or patterns are in a sequence and justifying the reasons.

NOTES

Emphasise use of $3n$ meaning $3 \times n$.
Students need to be clear on the description of the pattern in words, the difference between the terms and the algebraic description of the nth term.
Students are not expected to find the nth term of a quadratic sequence.