UNIT 7: Averages and range, sampling, collecting data, analysing data

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SPECIFICATION REFERENCES

S1 infer properties of populations or distributions from a sample, while knowing the limitations of sampling

S2 interpret and construct tables, charts and diagrams, including frequency tables, bar charts, pie charts and pictograms for categorical data, vertical line charts for ungrouped discrete numerical data, tables and line graphs for time–series data and know their appropriate use

S4 interpret, analyse and compare the distributions of data sets from univariate empirical distributions through: …

- appropriate measures of central tendency (median, mean, mode and modal class) and spread (range, including consideration of outliers)

PRIOR KNOWLEDGE

Students should be able to calculate the midpoint of two numbers.

Students will have drawn the statistical diagrams in unit 3.

Students will have used inequality notation.

KEYWORDS

Tier 2
Mean, range, outlier, frequency, bias, sample
Tier 3
Mode, median, midpoint

SMSC/RWCM/CEIAG
This topic is useful for any career involving analysing data and drawing conclusions from it. It would be needed for any statisticians role in jobs as varied as football analyst, doctor and town planner.

7a. Statistics and sampling (S1)	**Teaching time** 3–5 hours

OBJECTIVES

By the end of the sub-unit, students should be able to:

- Specify the problem and:
  - plan an investigation;
  - decide what data to collect and what statistical analysis is needed;
  - consider fairness;
- Recognise types of data: primary secondary, quantitative and qualitative;
- Identify which primary data they need to collect and in what format, including grouped data;
- Collect data from a variety of suitable primary and secondary sources;
- Understand how sources of data may be biased;
- Explain why a sample may not be representative of a whole population;
- Understand sample and population.

POSSIBLE SUCCESS CRITERIA/EXAM QUESTIONS

Explain why a sample may not be representative of a whole population.

Carry out a statistical investigation of their own and justify how sources of bias have been eliminated.

Show me an example of a situation in which biased data would result.

There are 1200 students at a school.

Kate is helping to organise a party.
She is going to order a pizza.

Kate takes a sample of 60 of the students at the school.
She asks each student to tell her **one** type of pizza they want.

The table shows information about her results.

Pizza	Number of students
ham	20
salami	15
vegetarian	8
margherita	17

Work out how much ham pizza Kate should order.
Write down any assumption you make **and** explain how this could affect your answer.

(Total 3 marks)

New SAMs Paper 1F qu.27 / 1H qu.7 (S1 – AO1/AO3)

OPPORTUNITIES FOR REASONING/PROBLEM SOLVING

When using a sample of a population to solve contextual problem, students should be able to justify why the sample may not be representative of the whole population.

COMMON MISCONCEPTIONS

The concept of an unbiased sample is difficult for some students to understand.

NOTES

Emphasise the difference between primary and secondary sources and remind students about the different between discrete and continuous data.

Discuss sample size and mention that a census is the whole population (the UK census takes place every 10 years in a year ending with a 1 – the next one is due in 2021).

Specify the problem and planning for data collection is not included in the programme of study but is a perquisite to understand the context of the topic.

Writing a questionnaire is not part of the new specification, but is a good topic to demonstrate bias and ways to reduce bias in terms of timing, location and question types that can introduce bias.

7b. The averages (S2, S4)	**Teaching time** 5–7 hours

OBJECTIVES

By the end of the sub-unit, students should be able to:

- Calculate the mean, mode, median and range for discrete data;
- Can interpret and find a range of averages as follows:
  - median, mean and range from a (discrete) frequency table;
  - range, modal class, interval containing the median, and estimate of the mean from a grouped data frequency table;
  - mode and range from a bar chart;
  - median, mode and range from stem and leaf diagrams;
  - mean from a bar chart;
- Understand that the expression 'estimate' will be used where appropriate, when finding the mean of grouped data using mid-interval values;
- Compare the mean, median, mode and range (as appropriate) of two distributions using bar charts, dual bar charts, pictograms and back-to-back stem and leaf;
- Recognise the advantages and disadvantages between measures of average.

POSSIBLE SUCCESS CRITERIA/EXAM QUESTIONS

State the median, mode, mean and range from a small data set.

Extract the averages from a stem and leaf diagram.

Estimate the mean from a table.

The table shows some information about the foot lengths of 40 adults.

Foot length (f cm)	Number of adults
$16 \leqslant f < 18$	3
$18 \leqslant f < 20$	6
$20 \leqslant f < 22$	10
$22 \leqslant f < 24$	12
$24 \leqslant f < 26$	9

(a) Write down the modal class interval.

(1)

(b) Calculate an estimate for the mean foot length.

(3)

(Total 4 marks)

New SAMs Paper 2F qu.27 / 2H qu.5 (S4 – AO1/AO2)

OPPORTUNITIES FOR REASONING/PROBLEM SOLVING

Students should be able to provide a correct solution as a counter-argument to statements involving the "averages", e.g. Susan states that the median is 15, she is wrong. Explain why.

Given the mean, median and mode of five positive whole numbers, can you find the numbers?

COMMON MISCONCEPTIONS

Often the $\sum(m \times f)$ is divided by the number of classes rather than $\sum f$ when estimating the mean.

NOTES

Encourage students to cross out the midpoints of each group once they have used these numbers to in $m \times f$. This helps students to avoid summing m instead of f.

Remind students how to find the midpoint of two numbers.

Emphasise that continuous data is measured, i.e. length, weight, and discrete data can be counted, i.e. number of shoes.

When comparing the mean and range of two distributions support with 'copy and complete' sentences, or suggested wording.

UNIT 4: Fractions and percentages 4: Fractions and percentages

Return to Overview

SPECIFICATION REFERENCES

N1 order positive and negative integers, decimals and fractions; use the symbols =, ≠, <, >, ≤, ≥

N2 apply the four operations, including formal written methods, to integers, decimals and simple fractions (proper and improper), and mixed numbers – all both positive and negative; understand and use place value (e.g. when working with very large or very small numbers, and when calculating with decimals)

N3 recognise and use relationships between operations, including inverse operations (e.g. cancellation to simplify calculations and expressions); use conventional notation for priority of operations, including brackets, powers, roots and reciprocals

N8 calculate exactly with fractions …

N10 work interchangeably with terminating decimals and their corresponding fractions (such as 3.5 and or 0.375 and)

N12 interpret fractions and percentages as operators

N13 use standard units of mass, length, time, money and other measures (including standard compound measures) using decimal quantities where appropriate

R3 express one quantity as a fraction of another, where the fraction is less than 1 or greater than 1

R9 define percentage as 'number of parts per hundred'; interpret percentages and percentage changes as a fraction or a decimal, and interpret these multiplicatively; express one quantity as a percentage of another; compare two quantities using percentages; work with percentages greater than 100%; solve problems involving percentage change, including percentage increase/decrease, and original value problems and simple interest including in financial mathematics

S2 interpret and construct tables, charts and diagrams, including frequency tables, bar charts, pie charts and pictograms for categorical data, vertical line charts for ungrouped discrete numerical data, tables and line graphs for time series data and know their appropriate use

PRIOR KNOWLEDGE

Students should be able to use the four operations of number.

Students should be able to find common factors.

Students have a basic understanding of fractions as being 'parts of a whole'.

Students should be able to define percentage as 'number of parts per hundred'.

Students should know number complements to 10 and multiplication tables.

KEYWORDS

Tier 2
Mixed, improper, increase, decrease
Tier 3
Decimal, percentage, inverse, denominator, numerator, terminating, recurring, multiplier

SMSC/RWCM/CEIAG

This topic is needed to help understand and manage finances including shopping in the sales, price increases, budgeting, loans and investments.

4a. Fractions (N1, N2, N3, N12, N13, R3, S2)	**Teaching time** 6–8 hours

OBJECTIVES

By the end of the sub-unit, students should be able to:

- Use diagrams to find equivalent fractions or compare fractions;
- Write fractions to describe shaded parts of diagrams;
- Express a given number as a fraction of another, using very simple numbers, some cancelling, and where the fraction is both < 1 and > 1;
- Write a fraction in its simplest form and find equivalent fractions;
- Order fractions, by using a common denominator;
- Compare fractions, use inequality signs, compare unit fractions;
- Convert between mixed numbers and improper fractions;
- Add and subtract fractions;
- Add fractions and write the answer as a mixed number;
- Multiply and divide an integer by a fraction;
- Multiply and divide a fraction by an integer, including finding fractions of quantities or measurements, and apply this by finding the size of each category from a pie chart using fractions;
- Understand and use unit fractions as multiplicative inverses;
- Multiply fractions: simplify calculations by cancelling first;
- Divide a fraction by a whole number;
- Divide fractions by fractions.

POSSIBLE SUCCESS CRITERIA/EXAM QUESTIONS

Express a given number as a fraction of another, including where the fraction > 1.

Simplify $\frac{120}{100}$.

$\frac{3}{5} \times 15$, $20 \times \frac{3}{4}$.

$\frac{1}{2}$ of 36 m, $\frac{1}{4}$ of £20.

Find the size of each category from a pie chart using fractions.

Calculate: $\frac{1}{2} \times \frac{6}{7}$, $\frac{3}{5} \div 3$.

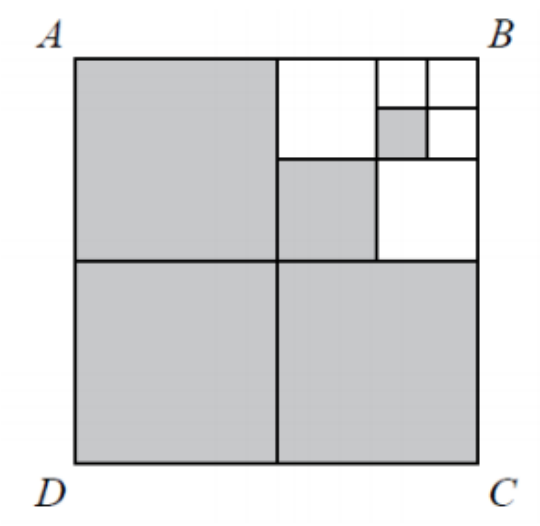
There are 6760 people at a rugby match.
3879 of the people are men.
1241 of the people are women.
$\frac{1}{4}$ of the children are girls.

Work out how many boys are at the rugby match.

(Total 3 marks)

New SAMs Paper 3F qu.3 (N2 – AO1/AO3)

ABCD is a square.
This diagram is drawn accurately



What fraction of the square *ABCD* is shaded?

(Total 2 marks)

New SAMs Paper 2F qu.7 (N8 – AO1/AO2)

COMMON MISCONCEPTIONS

The larger the denominator the larger the fraction.

NOTES

When expressing a given number as a fraction of another, start with very simple numbers < 1, and include some cancelling before fractions using numbers > 1.

When adding and subtracting fractions, start with same denominator, then where one denominator is a multiple of the other (answers ≤ 1), and finally where both denominators have to be changed (answers ≤ 1).
Regular revision of fractions is essential.
Demonstrate how to the use the fraction button on the calculator.
Use real-life examples where possible.

Use long division to illustrate recurring decimals.

4b. Fractions, decimals and percentages (N1, N2, N8, N10, N13, R9)	**Teaching time** 3–5 hours

OBJECTIVES

By the end of the sub-unit, students should be able to:

- Recall the fraction-to-decimal conversion;
- Convert between fractions and decimals;
- Convert a fraction to a decimal to make a calculation easier, e.g. $0.25 \times 8 = \frac{1}{4} \times 8$, or $\frac{3}{8} \times 10 = 0.375 \times 10$;
- Recognise recurring decimals and convert fractions such as $\frac{3}{7}$, $\frac{1}{3}$ and $\frac{2}{3}$ into recurring decimals;
- Compare and order fractions, decimals and integers, using inequality signs;
- Understand that a percentage is a fraction in hundredths;
- Express a given number as a percentage of another number;
- Convert between fractions, decimals and percentages;
- Order fractions, decimals and percentages, including use of inequality signs.

POSSIBLE SUCCESS CRITERIA

Write terminating decimals (up to 3 d.p.) as fractions.

Convert between fractions, decimals and percentages, common ones such as $\frac{1}{2}$, $\frac{1}{10}$, $\frac{1}{4}$, $\frac{3}{4}$ and $\frac{n}{10}$.

Order integers, decimals and fractions.

Here are four numbers.

$$0.43 \qquad \frac{3}{7} \qquad 43.8\% \qquad \frac{7}{16}$$

Write these numbers in order of size.
Start with the smallest number.

(Total 2 marks)

New SAMs Paper 3F qu.10 (N1, N10 – AO1)

OPPORTUNITIES FOR REASONING/PROBLEM SOLVING

Questions that involve rates of overtime pay including simple calculations involving fractional (>1, e.g. 1.5) and hourly pay. These can be extended into calculating rates of pay given the final payment and number of hours worked.

Working out the number of people/things where the number of people/things in different categories is given as a fraction, decimal or percentage.

COMMON MISCONCEPTIONS

Incorrect links between fractions and decimals, such as thinking that = 0.15, 5% = 0.5,

4% = 0.4, etc.

It is not possible to have a percentage greater than 100%.

NOTES

Students should be reminded of basic percentages and fraction conversions.

Emphasise the importance of being able to convert between fractions, decimals and percentages to make calculations easier.