UNIT 1: Powers, decimals, HCF and LCM, positive and negative, roots, rounding, reciprocals, standard form, indices and surds

SPECIFICATION REFERENCES

N2 apply the four operations, including formal written methods, to integers, decimals … both positive and negative; understand and use place value (e.g. working with very large or very small numbers, and when calculating with decimals)

N3 recognise and use relationships between operations, including inverse operations (e.g. cancellation to simplify calculations and expressions); use conventional notation for priority of operations, including brackets, powers, roots and reciprocals

N4 use the concepts and vocabulary of prime numbers, factors (divisors), multiples, common factors, common multiples, highest common factor, lowest common multiple, prime factorisation, including using product notation and the unique factorisation theorem

N5 apply systematic listing strategies **including use of the product rule for counting (i.e. if there are m ways of doing one task and for each of these, there are n ways of doing another task, then the total number of ways the two tasks can be done is $m \times n$ ways)**

N6 use positive integer powers and associated real roots (square, cube and higher), recognise powers of 2, 3, 4, 5; **estimate powers and roots of any given positive number**

N7 calculate with roots and with integer **and fractional** indices

N8 calculate exactly with … **surds**; … **simplify surd expressions involving squares (e.g. $\sqrt{12} = \sqrt{(4 \times 3)} = \sqrt{4} \times \sqrt{3} = 2\sqrt{3}$)**

N9 calculate with and interpret standard form $A \times 10^n$, where $1 \leq A < 10$ and n is an integer.

N14 estimate answers; check calculations using approximation and estimation, including answers obtained using technology

N15 round numbers and measures to an appropriate degree of accuracy (e.g. to a specified number of decimal places or significant figures); …

PRIOR KNOWLEDGE

It is essential that students have a firm grasp of place value and be able to order integers and decimals and use the four operations.
Students should have knowledge of integer complements to 10 and to 100, multiplication facts to 10 × 10, strategies for multiplying and dividing by 10, 100 and 1000.
Students will have encountered squares, square roots, cubes and cube roots and have knowledge of classifying integers.

KEYWORDS

Tier 2
Sum, product, division, rounding, significant, factor, multiple, prime, square, cube, power, digit, negative, remainder, operation, root, even, odd
Tier 3
Placeholder, BIDMAS, decimal place, LCM, HCF, index notation, integer, estimate, surd, rational, standard form

SMSC/RWCM/CEIAG
Useful for any career where budgeting is required, calculations are needed or understanding of approximation – engineering, accountancy, decorator, construction are all examples of this

1a. Calculations, checking and rounding	**Teaching time**
(N2, N3, N5, N14, N15)	6–8 hours

OBJECTIVES

By the end of the sub-unit, students should be able to:

- Add, subtract, multiply and divide decimals and whole numbers;
- Multiply or divide by any number between 0 and 1;
- Put digits in the correct place in a decimal calculation and use one calculation to find the answer to another;
- Use the product rule for counting (i.e. if there are m ways of doing one task and for each of these, there are n ways of doing another task, then the total number of ways the two tasks can be done is $m \times n$ ways);
- Round numbers to the nearest 10, 100, 1000;
- Round to the nearest integer, to a given number of decimal places and to a given number of significant figures;
- Estimate answers to one- or two-step calculations, including use of rounding numbers and formal estimation to 1 significant figure: mainly whole numbers and then decimals.

POSSIBLE SUCCESS CRITERIA/EXAM QUESTIONS

Given 5 digits, what is the largest even number, largest odd number, or largest or smallest answers when subtracting a two-digit number from a three-digit number?
Given 2.6 × 15.8 = 41.08 what is 26 × 0.158? What is 4108 ÷ 26?

Marie has 25 cards.
Each card has a different symbol on it.

Marie gives one card to Shelley and one card to Pauline.

(*a*) In how many different ways can Marie do this?

(2)

There are 12 boys and 10 girls in David's class.
David is going to pick three different students from his class and write their names in a list in order.

The order will be

boy girl boy	**or**	girl boy girl

(*b*) How many different lists can David write?

(3)

(Total 5 marks)

Specimen Papers Set 2, Paper 2H qu.12 (N5 – AO1/AO3)

There are 95 girls and 87 boys in Year 13 at a school.

One girl is going to be chosen for the role of Head Girl.
A different girl is going to be chosen for the role of Deputy Head Girl.
One boy is going to be chosen for the role of Head Boy.
A different boy is going to be chosen for the role of Deputy Head Boy.

Work out how many different ways this can be done.

(Total 3 marks)

Mock Papers Set 1, Paper 2H qu.18 (N5 – AO1)

Kiera used her calculator to work out the value of a number x.
She wrote down the first two digits of the answer on her calculator.

She wrote down 7.3

Write down the error interval for x.

(Total 2 marks)

Mock Papers Set 3, Paper 3H qu.8 (N15 – AO1)

OPPORTUNITIES FOR REASONING/PROBLEM SOLVING

Problems that include providing reasons as to whether an answer is an overestimate or underestimate.
Missing digits in calculations involving the four operations.
Questions such as: Phil states 3.44 × 10 = 34.4, and Chris states 3.44 × 10 = 34.40. Who is correct?
Show me another number with 3, 4, 5, 6, 7 digits that includes a 6 with the same value as the "6" in the following number 36 754.

COMMON MISCONCEPTIONS

Significant figure and decimal place rounding are often confused.
Some pupils may think 35 934 = 36 to two significant figures.

NOTES

The expectation for Higher tier is that much of this work will be reinforced throughout the course.
Particular emphasis should be given to the importance of clear presentation of work.
Formal written methods of addition, subtraction and multiplication work from right to left, whilst formal division works from left to right.
Any correct method of multiplication will still gain full marks, for example, the grid method, the traditional method, Napier's bones.
Encourage the exploration of different calculation methods.
Amounts of money should always be rounded to the nearest penny.
Make sure students are absolutely clear about the difference between significant figures and decimal places.

1b. Indices, roots, reciprocals and hierarchy of operations	**Teaching time**
(N2, N3, N6, N7, N14)	7–9 hours

OBJECTIVES

By the end of the sub-unit, students should be able to:

- Use index notation for integer powers of 10, including negative powers;
- Recognise powers of 2, 3, 4, 5;
- Use the square, cube and power keys on a calculator and estimate powers and roots of any given positive number, by considering the values it must lie between, e.g. the square root of 42 must be between 6 and 7;
- Find the value of calculations using indices including positive, fractional and negative indices;
- Recall that $n^0 = 1$ and $n^{-1} = \frac{1}{n}$ for positive integers n as well as, $n^{\frac{1}{2}} = \sqrt{n}$ and $n^{\frac{1}{3}} = \sqrt[3]{n}$ for any positive number n;
- Understand that the inverse operation of raising a positive number to a power n is raising the result of this operation to the power $\frac{1}{n}$;
- Use index laws to simplify and calculate the value of numerical expressions involving multiplication and division of integer powers, fractional and negative powers, and powers of a power;
- Solve problems using index laws;
- Use brackets and the hierarchy of operations up to and including with powers and roots inside the brackets, or raising brackets to powers or taking roots of brackets;
- Use an extended range of calculator functions, including +, −, ×, ÷, x^2, $\sqrt{x}$, memory, x^y, $x^{\frac{1}{y}}$, brackets;
- Use calculators for all calculations: positive and negative numbers, brackets, powers and roots, four operations.

POSSIBLE SUCCESS CRITERIA/EXAM QUESTIONS

What is the value of 2^5?
Prove that the square root of 45 lies between 6 and 7.
Evaluate $(2^3 \times 2^5) \div 2^4$, 4^0, $8^{-\frac{2}{3}}$.
Work out the value of n in $40 = 5 \times 2^n$.

Work out the reciprocal of 0.125.

(Total 1 mark)

Specimen Papers Set 1, Paper 3F qu.3 (N3 – AO1)

(*a*) Write down the value of $64^{\frac{1}{2}}$

(1)

(*b*) Find the value of $\left(\frac{8}{125}\right)^{-\frac{2}{3}}$

(2)

(Total 3 marks)

New SAMs Paper 1H qu.10 (N7, N8 – AO1)

(i) Find the value of $\sqrt[5]{3.2 \times 10^{11}}$

(ii) Find the value of $10^{\frac{3}{4}}$
Give your answer correct to 1 decimal place.

(Total 2 marks)

Specimen Papers Set 2, Paper 2H qu.16 (N7, N9 – AO1)

OPPORTUNITIES FOR REASONING/PROBLEM SOLVING

Problems that use indices instead of integers will provide rich opportunities to apply the knowledge in this unit in other areas of Mathematics.

COMMON MISCONCEPTIONS

The order of operations is often not applied correctly when squaring negative numbers, and many calculators will reinforce this misconception.

NOTES

Students need to know how to enter negative numbers into their calculator.
Use negative number and not minus number to avoid confusion with calculations.

1c. Factors, multiples and primes	**Teaching time**
(N3, N4)	5–7 hours

OBJECTIVES

By the end of the sub-unit, students should be able to:

- Identify factors, multiples and prime numbers;
- Find the prime factor decomposition of positive integers – write as a product using index notation;
- Find common factors and common multiples of two numbers;
- Find the LCM and HCF of two numbers, by listing, Venn diagrams and using prime factors – include finding LCM and HCF given the prime factorisation of two numbers;
- Solve problems using HCF and LCM, and prime numbers;
- Understand that the prime factor decomposition of a positive integer is unique, whichever factor pair you start with, and that every number can be written as a product of prime factors.

POSSIBLE SUCCESS CRITERIA/EXAM QUESTIONS

Know how to test if a number up to 120 is prime.
Understand that every number can be written as a unique product of its prime factors.
Recall prime numbers up to 100.
Understand the meaning of prime factor.
Write a number as a product of its prime factors.
Use a Venn diagram to sort information.

Liz buys packets of coloured buttons.

There are 8 red buttons in each packet of red buttons.
There are 6 silver buttons in each packet of silver buttons.
There are 5 gold buttons in each packet of gold buttons.

Liz buys equal numbers of red buttons, silver buttons and gold buttons.
How many packets of each colour of buttons did Liz buy?

(Total 3 marks)

New SAMs Paper 3H qu.6 (N4 – AO1/AO3)

OPPORTUNITIES FOR REASONING/PROBLEM SOLVING

Evaluate statements and justify which answer is correct by providing a counter-argument by way of a correct solution.

COMMON MISCONCEPTIONS

1 is a prime number.
Particular emphasis should be made on the definition of "product" as multiplication, as many students get confused and think it relates to addition.

NOTES

Use a number square to find primes (Eratosthenes sieve).
Using a calculator to check the factors of large numbers can be useful.
Students need to be encouraged to learn squares from 2 × 2 to 15 × 15 and cubes of 2, 3, 4, 5 and 10, and corresponding square and cube roots.

1d. Standard form and surds	**Teaching time**
(N8, N9)	5–7 hours

OBJECTIVES

By the end of the sub-unit, students should be able to:

- Convert large and small numbers into standard form and vice versa;
- Add and subtract numbers in standard form;
- Multiply and divide numbers in standard form;
- Interpret a calculator display using standard form and know how to enter numbers in standard form;
- Understand surd notation, e.g. calculator gives answer to sq rt 8 as 4 rt 2;
- Simplify surd expressions involving squares (e.g. $\sqrt{12} = \sqrt{(4 \times 3)} = \sqrt{4} \times \sqrt{3} = 2\sqrt{3}$).

POSSIBLE SUCCESS CRITERIA/EXAM QUESTIONS

Write 51080 in standard form.
Write 3.74×10^{-6} as an ordinary number.
Simplify $\sqrt{8}$.
Convert a 'near miss', or any number, into standard form; e.g. 23×10^{7}.

1. Number

Show that $\dfrac{1}{1+\dfrac{1}{\sqrt{2}}}$ can be written as $2 - \sqrt{2}$

(Total 3 marks)
New SAMs Paper 1H qu.23 (N8 – AO2)

One uranium atom has a mass of 3.95×10^{-22} grams.

(*a*) Work out an estimate for the number of uranium atoms in 1 kg of uranium.

(3)

(*b*) Is your answer to (*a*) an underestimate or an overestimate?
Give a reason for your answer.

(1)

(Total 4 marks)
New SAMs Paper 1H qu.11 (N9, N14, R1, R10 – AO1/AO3)

OPPORTUNITIES FOR REASONING/PROBLEM SOLVING

Links with other areas of Mathematics can be made by using surds in Pythagoras and when using trigonometric ratios.

COMMON MISCONCEPTIONS

Some students may think that any number multiplied by a power of ten qualifies as a number written in standard form.
When rounding to significant figures some students may think, for example, that 6729 rounded to one significant figure is 7.

NOTES

Standard form is used in science and there are lots of cross-curricular opportunities.
Students need to be provided with plenty of practice in using standard form with calculators.
Rationalising the denominator is covered later in unit 17.