UNIT 2: Expressions, substituting into simple formulae, expanding and factorising, equations, sequences and inequalities, simple proof

SPECIFICATION REFERENCES

N1 … use the symbols =, ≠, <, >, ≤, ≥

N3 recognise and use relationships between operations, including inverse operations (e.g. cancellation to simplify calculations and expressions); use conventional notation for priority of operations, including brackets, powers, roots and reciprocals

N8 calculate exactly with fractions, **surds** …; **simplify surd expressions involving squares** …

N9 calculate with and interpret standard form $A \times 10^n$, where $1 \leq A < 10$ and n is an integer.

A1 use and interpret algebraic notation, including:

- ab in place of $a \times b$
- $3y$ in place of $y + y + y$ and $3 \times y$
- a^2 in place of $a \times a$, a^3 in place of $a \times a \times a$, a^2b in place of $a \times a \times b$
- $\frac{a}{b}$ in place of $a \div b$
- coefficients written as fractions rather than as decimals
- brackets

A2 substitute numerical values into formulae and expressions, including scientific formulae

A3 understand and use the concepts and vocabulary of expressions, equations, formulae, identities, inequalities, terms and factors

A4 simplify and manipulate algebraic expressions … by:

- collecting like terms
- multiplying a single term over a bracket
- taking out common factors
- expanding products of two … binomials
- factorising quadratic expressions of the form $x^2 + bx + c$, including the difference of two squares; …
- simplifying expressions involving sums, products and powers, including the laws of indices

A5 understand and use standard mathematical formulae; rearrange formulae to change the subject

A6 know the difference between an equation and an identity; argue mathematically to show algebraic expressions are equivalent, and use algebra to support and construct arguments **and proofs**

A7 where appropriate, interpret simple expressions as functions with inputs and outputs; …

A17 solve linear equations in one unknown algebraically …;

A20 **find approximate solutions to equations numerically using iteration**

A21 translate simple situations or procedures into algebraic expressions or formulae; derive an equation …, solve the equation and interpret the solution

A23 generate terms of a sequence from either a term-to-term or a position-to-term rule

A24 recognise and use sequences of triangular, square and cube numbers, simple arithmetic progressions, Fibonacci type sequences and simple geometric progressions (r^n where n is an integer, and r is a rational number > 0), recognise and use other sequences **or a surd)**

A25 deduce expressions to calculate the nth term of linear sequences.

PRIOR KNOWLEDGE

Students should have prior knowledge of some of these topics, as they are encountered at Key Stage 3:

- the ability to use negative numbers with the four operations and recall and use hierarchy of operations and understand inverse operations;
- dealing with decimals and negatives on a calculator;
- using index laws numerically.

KEYWORDS

Tier 2
Expression, term, variable, simplify, factor, term, identity, power
Tier 3
Formula, factorise, equation, substitute, index, expand, bracket, linear, fractional, quadratic, arithmetic, geometric, derive, nth term

SMSC/RWCM/CEIAG
Any medical career will use substitution to ensure correct dosages of medicines for different patients.
Sequences are used in lots of analytical jobs to help identify patterns

2a. Algebra: the basics	**Teaching time**
(N1, N3, A1, A2, A3, A4, A7)	7–9 hours

OBJECTIVES

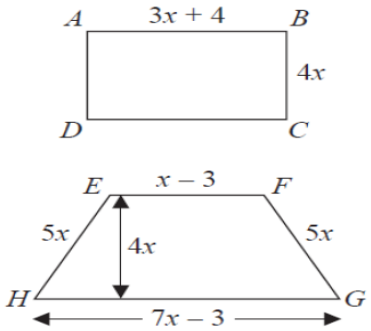
By the end of the sub-unit, students should be able to:

- Use algebraic notation and symbols correctly;
- Write an expression;
- Know the difference between a term, expression, equation, formula and an identity;
- Manipulate an expression by collecting like terms;
- Substitute positive and negative numbers into expressions such as $3x + 4$ and $2x^3$ and then into expressions involving brackets and powers;
- Substitute numbers into formulae from mathematics and other subject using simple linear formulae, e.g. $l \times w$, $v = u + at$;
- Simplify expressions by cancelling, e.g. $\frac{4x}{2} = 2x$
- Use instances of index laws for positive integer powers;
- Use index notation (positive powers) when multiplying or dividing algebraic terms;
- Use instances of index laws, including use of zero, fractional and negative powers;
- Multiply a single term over a bracket;
- Recognise factors of algebraic terms involving single brackets and simplify expressions by factorising, including subsequently collecting like terms;
- Expand the product of two linear expressions, i.e. double brackets working up to negatives in both brackets and also similar to $(2x + 3y)(3x - y)$;
- Know that squaring a linear expression is the same as expanding double brackets;
- Factorise quadratic expressions of the form $ax^2 + bx + c$;
- Factorise quadratic expressions using the difference of two squares.

POSSIBLE SUCCESS CRITERIA/EXAM QUESTIONS

Simplify $4p - 2q^2 + 1 - 3p + 5q^2$.
Evaluate $4x^2 - 2x$ when $x = -5$.
Simplify $z^4 \times z^3$, $y^3 \div y^2$, $(a^7)^2$, $\left(8x^6y^4\right)^{\frac{1}{3}}$.
Expand and simplify $3(t - 1) + 57$.
Factorise $15x^2y - 35x^2y^2$.
Expand and simplify $(3x + 2)(4x - 1)$.
Factorise $6x^2 - 7x + 1$.

ABCD is a rectangle.
EFGH is a trapezium.



All measurements are in centimetres.
The perimeters of these two shapes are the same.

Work out the area of the rectangle.

(Total 5 marks)

New SAMs Paper 2H qu.9 (A2, A4, A17, A21 – AO1/AO3)

Show that

$$(3x - 1)(x + 5)(4x - 3) = 12x^3 + 47x^2 - 62x + 15$$

for all values of x.

(Total 3 marks)

Specimen Papers Set 1, Paper 2H qu.13 (A4, A6 – AO2)

Martin expands $(2x + 1)(2x - 3)(3x + 2)$

He gets $12x^3 - 4x^2 - 17x + 6$

Explain why Martin's solution cannot be correct.

(Total 1 mark)

Mock Papers Set 1, Paper 1H qu.16b (A4 – AO3)

OPPORTUNITIES FOR REASONING/PROBLEM SOLVING

Evaluate statements and justify which answer is correct by providing a counter-argument by way of a correct solution.

COMMON MISCONCEPTIONS

When expanding two linear expressions, poor number skills involving negatives and times tables will become evident.

NOTES

Some of this will be a reminder from Key Stage 3 and could be introduced through investigative material such as handshake, frogs etc.
Students will have encountered much of this before and you may wish to introduce solving equations using function machines.
Practise factorisation where more than one variable is involved. NB More complex quadratics are covered in a later unit.
Plenty of practice should be given for factorising, and reinforce the message that making mistakes with negatives and times tables is a different skill to that being developed. Encourage students to expand linear sequences prior to simplifying when dealing with "double brackets".

2b. Setting up, rearranging and solving equations	**Teaching time**
(N1, N8, A1, A2, A5, A6, A17, A20, A21)	7–9 hours

OBJECTIVES

By the end of the sub-unit, students should be able to:

- Set up simple equations from word problems and derive simple formulae;
- Understand the ≠ symbol (not equal), e.g. $6x + 4 \neq 3(x + 2)$, and introduce identity ≡ sign;
- Solve linear equations, with integer coefficients, in which the unknown appears on either side or on both sides of the equation;
- Solve linear equations which contain brackets, including those that have negative signs occurring anywhere in the equation, and those with a negative solution;
- Solve linear equations in one unknown, with integer or fractional coefficients;
- Set up and solve linear equations to solve to solve a problem;
- Derive a formula and set up simple equations from word problems, then solve these equations, interpreting the solution in the context of the problem;
- Substitute positive and negative numbers into a formula, solve the resulting equation including brackets, powers or standard form;
- Use and substitute formulae from mathematics and other subjects, including the kinematics formulae $v = u + at$, $v^2 - u^2 = 2as$, and $s = ut + \frac{1}{2}at^2$;
- Change the subject of a simple formula, i.e. linear one-step, such as $x = 4y$;
- Change the subject of a formula, including cases where the subject is on both sides of the original formula, or involving fractions and small powers of the subject;
- Simple proofs and use of ≡ in "show that" style questions; know the difference between an equation and an identity;
- Use iteration to find approximate solutions to equations, for simple equations in the first instance, then quadratic and cubic equations.

POSSIBLE SUCCESS CRITERIA/EXAM QUESTIONS

A room is 2 m longer than it is wide. If its area is 30 m^2 what is its perimeter?
Use fractions when working in algebraic situations.
Substitute positive and negative numbers into formulae.
Be aware of common scientific formulae.
Know the meaning of the 'subject' of a formula.
Change the subject of a formula when one step is required.
Change the subject of a formula when two steps are required.

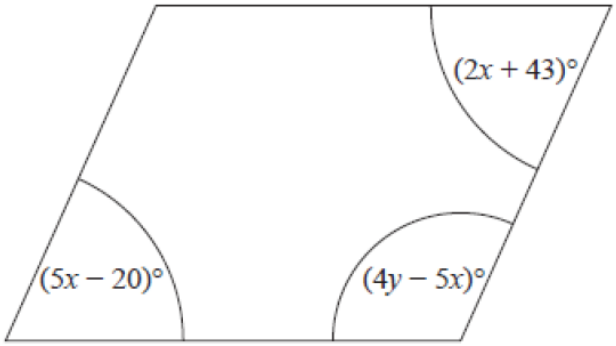
Make a the subject of $a + 3 = \frac{2a + 7}{r}$

(Total 3 marks)
New SAMs Paper 1H qu.17 (A5 – AO1)

Solve $\frac{x + 2}{3x} + \frac{x - 2}{2x} = 3$

(Total 3 marks)
Specimen Papers Set 1, Paper 1H qu.14 (A17 –AO1)

Here is a parallelogram.



Work out the value of x and the value of y.

(Total 5 marks)

New SAMs Paper 1F qu.28 / 1H qu.8 (A21, G4 – AO1/AO3)

OPPORTUNITIES FOR REASONING/PROBLEM SOLVING

Forming and solving equations involving algebra and other areas of mathematics such as area and perimeter.
Evaluate statements and justify which answer is correct by providing a counter-argument by way of a correct solution.

COMMON MISCONCEPTIONS

Hierarchy of operations applied in the wrong order when changing the subject of a formula.
$a^0 = 0$.
$3xy$ and $5yx$ are different "types of term" and cannot be "collected" when simplifying expressions.
The square and cube operations on a calculator may not be similar on all makes.
Not using brackets with negative numbers on a calculator.
Not writing down all the digits on the display.

NOTES

Emphasise good use of notation.
Students need to realise that not all linear equations can be solved by observation or trial and improvement, and hence the use of a formal method is important.
Students can leave their answer in fraction form where appropriate. Emphasise that fractions are more accurate in calculations than rounded percentage or decimal equivalents.
Use examples involving formulae for circles, spheres, cones and kinematics when changing the subject of a formula.
For substitution use the distance–time–speed formula, and include speed of light given in standard form.
Students should be encouraged to use their calculator effectively by using the replay and ANS/EXE functions; reinforce the use of brackets and only rounding their final answer with trial and improvement.

2c. Sequences (N8, N9, A23, A24, A25)	**Teaching time** 5–7 hours

OBJECTIVES

By the end of the sub-unit, students should be able to:

- Recognise simple sequences including at the most basic level odd, even, triangular, square and cube numbers and Fibonacci-type sequences (including those involving numbers in standard form or index form);
- Generate sequences of numbers, squared integers and sequences derived from diagrams;
- Describe in words a term-to-term sequence and identify which terms cannot be in a sequence;
- Generate specific terms in a sequence using the position-to-term rule and term-to-term rule;
- Find and use (to generate terms) the nth term of an arithmetic sequence;
- Use the nth term of an arithmetic sequence to decide if a given number is a term in the sequence, or find the first term above or below a given number;
- Identify which terms cannot be in a sequence by finding the nth term;
- Continue a quadratic sequence and use the nth term to generate terms;
- Find the nth term of quadratic sequences;
- Distinguish between arithmetic and geometric sequences;
- Use finite/infinite and ascending/descending to describe sequences;
- Recognise and use simple geometric progressions (rn where n is an integer, and r is a rational number > 0 or a surd);
- Continue geometric progression and find term to term rule, including negative, fraction and decimal terms;
- Solve problems involving sequences from real life situations.

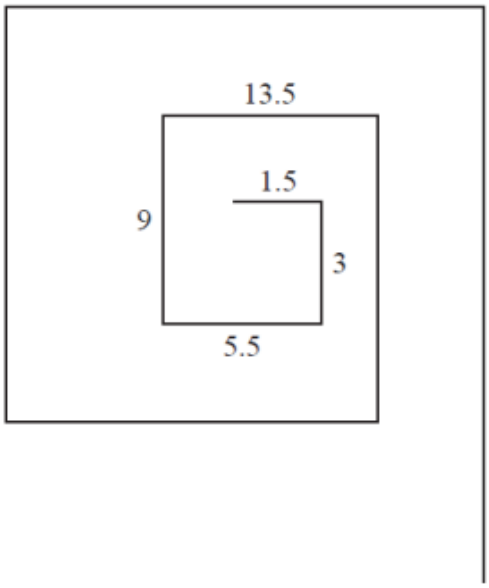
POSSIBLE SUCCESS CRITERIA/EXAM QUESTIONS

Given a sequence, 'which is the 1st term greater than 50?'

Be able to solve problems involving sequences from real-life situations, such as:

- 1 grain of rice on first square, 2 grains on second, 4 grains on third, etc (geometric progression), or person saves £10 one week, £20 the next, £30 the next, etc;
- What is the amount of money after x months saving the same amount, or the height of tree that grows 6 m per year;
- Compare two pocket money options, e.g. same number of £ per week as your age from 5 until 21, or starting with £5 a week aged 5 and increasing by 15% a year until 21.

The diagram shows the first 10 sides of a spiral pattern.
It also gives the lengths, in cm, of the first 5 sides.



The lengths, in cm, of the sides of the spiral form a sequence.

Find an expression in terms of n for the length, in cm, of the nth side.

(Total 3 marks)

Mock Papers Set 1, Paper 1H qu.18 (A25 – AO2/AO3)

Here are the first four terms of a quadratic sequence.

3 8 15 24

(*a*) Find an expression, in terms of n, for the nth term of this sequence.

(3)

The nth term of a different sequence is $2^n + 5$

(*b*) Show that 36 is **not** a term of this sequence.

(1)

(Total 4 marks)

Mock Papers Set 1, Paper 2H qu.12 (A24, A25 – AO1/AO2)

Here are the first six terms of a Fibonacci sequence.

1 1 2 3 5 8

The rule to continue a Fibonacci sequence is,

the next term in the sequence is the sum of the two previous terms.

(*a*) Find the 9th term of this sequence.

(1)

The first three terms of a different Fibonacci sequence are

a b $a + b$

(*b*) Show that the 6th term of this sequence is $3a + 5b$

(2)

Given that the 3rd term is 7 and the 6th term is 29,

(*c*) find the value of a and the value of b.

(3)

(Total 6 marks)

New SAMs Paper 3F qu.20 / 3H qu.3 (A24, A19, A21 – AO1/AO2/AO3)

OPPORTUNITIES FOR REASONING/PROBLEM SOLVING

Evaluate statements about whether or not specific numbers or patterns are in a sequence and justify the reasons.

COMMON MISCONCEPTIONS

Students struggle to relate the position of the term to "n".

NOTES

Emphasise use of $3n$ meaning $3 \times n$.

Students need to be clear on the description of the pattern in words, the difference between the terms and the algebraic description of the nth term.