UNIT 9: Algebra: Solving quadratic equations and inequalities, solving simultaneous equations algebraically

SPECIFICATION REFERENCES

N1 order positive and negative integers, decimals and fractions; use the symbols =, ≠, <, >, ≤, ≥

N8 calculate exactly with … **surds**; … **simplify surd expressions involving squares**
(e.g. $\sqrt{12} = \sqrt{(4 \times 3)} = \sqrt{4} \times \sqrt{3} = 2\sqrt{3}$)

A4 simplify and manipulate algebraic expressions (including those involving surds …) by: … **factorising quadratic expressions of the form $ax^2 + bx + c$**

A5 understand and use standard mathematical formulae; rearrange formulae to change the subject

A9 … find the equation of the line through two given points, or through one point with a given gradient

A11 identify and interpret roots … of quadratic functions algebraically …

A18 solve quadratic equations **(including those that require rearrangement)** algebraically by factorising, **by completing the square and by using the quadratic formula**; …

A19 solve two simultaneous equations in two variables (linear/linear **or linear/quadratic**) algebraically; find approximate solutions using a graph

A21 … derive an equation (or two simultaneous equations), solve the equation(s) and interpret the solution.

A22 solve linear inequalities in one **or two** variable**(s)**, **and quadratic inequalities in one variable**; represent the solution set on a number line, **using set notation and on a graph**

PRIOR KNOWLEDGE

Students should understand the ≥ and ≤ symbols.

Students can substitute into, solve and rearrange linear equations.

Students should be able to factorise simple quadratic expressions.

Students should be able to recognise the equation of a circle.

KEYWORDS

Tier 2
Solution, root, linear, solve, simultaneous, inequality, rearrange

Tier 3
Quadratic, completing the square, factorise

9a. Solving quadratics and simultaneous equations (N8, A4, A5, A9, A11, A18, A19, A21)	**Teaching time** 7–9 hours

OBJECTIVES

By the end of the sub-unit, students should be able to:

- Factorise quadratic expressions in the form $ax^2 + bx + c$;
- Solve quadratic equations by factorisation and completing the square;
- Solve quadratic equations that need rearranging;
- Set up and solve quadratic equations;
- Solve quadratic equations by using the quadratic formula;
- Find the exact solutions of two simultaneous equations in two unknowns;
- Use elimination or substitution to solve simultaneous equations;
- Solve exactly, by elimination of an unknown, two simultaneous equations in two unknowns:
  - linear / linear, including where both need multiplying;
  - linear / quadratic;
  - linear / $x^2 + y^2 = r^2$;
- Set up and solve a pair of linear simultaneous equations in two variables, including to represent a situation;
- Interpret the solution in the context of the problem;

POSSIBLE SUCCESS CRITERIA/EXAM QUESTIONS

Solve $3x^2 + 4 = 100$.

Know that the quadratic formula can be used to solve all quadratic equations, and often provides a more efficient method than factorising or completing the square.

Have an understanding of solutions that can be written in surd form.

(*a*) Write $2x^2 + 16x + 35$ in the form $a(x + b)^2 + c$ where a, b, and c are integers.

(3)

(*b*) Hence, or otherwise, write down the coordinates of the turning point of the graph of $y = 2x^2 + 16x + 35$

(1)

(Total 4 marks)

Specimen Papers Set 2, Paper 3H qu.23 (A11 – AO1)

Solve $x^2 - 6x - 8 = 0$
Write your answer in the form $a \pm \sqrt{b}$ where a and b are integers.

(Total 3 marks)
Specimen Papers Set 2, Paper 1H qu.17 (A18 –AO2)

Steve is asked to solve the equation $5(x + 2) = 47$
Here is his working.

$$\begin{aligned} 5(x + 2) &= 47 \\ 5x + 2 &= 47 \\ 5x &= 45 \\ x &= 9 \end{aligned}$$

Steve's answer is wrong.

(*a*) What mistake did he make?

(1)

Liz is asked to solve the equation $3x^2 + 8 = 83$

Here is her working.

$$\begin{aligned} 3x^2 + 8 &= 83 \\ 3x^2 &= 75 \\ x^2 &= 25 \\ x &= 5 \end{aligned}$$

(*b*) Explain what is wrong with Liz's answer.

(1)

(Total 2 marks)
Specimen Papers Set 2, Paper 2H qu.8 (A18, A17 –AO2/AO3)

Solve algebraically the simultaneous equations

$$\begin{aligned} x^2 + y^2 &= 25 \\ y - 2x &= 5 \end{aligned}$$

(Total 5 marks)
New SAMs Paper 2H qu.20 (A19 – AO1/AO2)

OPPORTUNITIES FOR REASONING/PROBLEM SOLVING

Problems that require students to set up and solve a pair of simultaneous equations in a

real-life context, such as 2 adult tickets and 1 child ticket cost £28, and 1 adult ticket and 3 child tickets cost £34. How much does 1 adult ticket cost?

COMMON MISCONCEPTIONS

Using the formula involving negatives can result in incorrect answers.

If students are using calculators for the quadratic formula, they can come to rely on them and miss the fact that some solutions can be left in surd form.

NOTES

Remind students to use brackets for negative numbers when using a calculator, and remind them of the importance of knowing when to leave answers in surd form.

Link to unit 2, where quadratics were solved algebraically (when $a = 1$).

The quadratic formula must now be known; it will not be given in the exam paper.

Reinforce the fact that some problems may produce one inappropriate solution which can be ignored.

Clear presentation of working out is essential.

Link with graphical representations.

9b. Inequalities (N1, A22)	**Teaching time** 5–7 hours

OBJECTIVES

By the end of the sub-unit, students should be able to:

- Show inequalities on number lines;
- Write down whole number values that satisfy an inequality;
- Solve simple linear inequalities in one variable, and represent the solution set on a number line;
- Solve two linear inequalities in x, find the solution sets and compare them to see which value of x satisfies both
  solve linear inequalities in two variables algebraically;
- Use the correct notation to show inclusive and exclusive inequalities.

POSSIBLE SUCCESS CRITERIA/EXAM QUESTIONS

Use inequality symbols to compare numbers.

Given a list of numbers, represent them on a number line using the correct notation.

Solve equations involving inequalities.

(*a*) Factorise $y^2 + 7y + 6$

(2)

(*b*) Solve $6x + 4 > x + 17$

(2)

(*c*) n is an integer with $-5 < 2n \leq 6$
Write down all the values of n

(2)

(Total 6 marks)
New SAMs Paper 3H qu.9 (A22, A4, N1 – AO1)

Solve $2x^2 - 5x - 12 > 0$

(Total 3 marks)
Mock Papers Set 2, Paper 3H qu.19 (A22 – AO1)

Solve the inequality $x^2 > 3(x + 6)$

(Total 4 marks)
Specimen Papers Set 2, Paper 1H qu.21 (A22 –AO1)

OPPORTUNITIES FOR REASONING/PROBLEM SOLVING

Problems that require student to justify why certain values in a solution can be ignored.

COMMON MISCONCEPTIONS

When solving inequalities students often state their final answer as a number quantity, and exclude the inequality or change it to =.

Some students believe that −6 is greater than −3.

NOTES

Emphasise the importance of leaving their answer as an inequality (and not changing it to =).

Link to units 2 and 9a, where quadratics and simultaneous equations were solved.
Students can leave their answers in fractional form where appropriate.
Ensure that correct language is used to avoid reinforcing misconceptions: for example, 0.15 should never be read as ‘zero point fifteen’, and $5 > 3$ should be read as ‘five is greater than 3’, not ‘5 is bigger than 3’

UNIT 4: Fractions, percentages, ratio and proportion

SPECIFICATION REFERENCES

N1 order positive and negative integers, decimals and fractions; …

N2 apply the four operations, including formal written methods, to integers, decimals and simple fractions (proper and improper), and mixed numbers – all both positive and negative; …

N3 recognise and use relationships between operations, including inverse operations (e.g. cancellation to simplify calculations and expressions); use conventional notation for priority of operations, including brackets, powers, roots and reciprocals

N8 calculate exactly with fractions …

N10 work interchangeably with terminating decimals and their corresponding fractions (such as 3.5 and or 0.375 and); **change recurring decimals into their corresponding fractions and vice versa**

N11 identify and work with fractions in ratio problems

N12 interpret fractions and percentages as operators

N13 use standard units of mass, length, time, money and other measures (including standard compound measures) using decimal quantities where appropriate

R2 use scale factors, scale diagrams and maps

R3 express one quantity as a fraction of another, where the fraction is less than 1 or greater than 1

R4 use ratio notation, including reduction to simplest form

R5 divide a given quantity into two parts in a given part:part or whole:part ratio; express the division of a quantity into two parts as a ratio; apply ratio to real contexts and problems (such as those involving conversion, comparison, scaling, mixing, concentrations)

R6 express a multiplicative relationship between two quantities as a ratio or a fraction

R7 understand and use proportion as equality of ratios

R8 relate ratios to fractions and to linear functions

R9 define percentage as 'number of parts per hundred'; interpret percentages and percentage changes as a fraction or a decimal, and interpret these multiplicatively; express one quantity as a percentage of another; compare two quantities using percentages; work with percentages greater than 100%; solve problems involving percentage change, including percentage increase/decrease, and original value problems and simple interest including in financial mathematics

R10 solve problems involving direct proportion; …

PRIOR KNOWLEDGE

Students should know the four operations of number.

Students should be able to find common factors.

Students should have a basic understanding of fractions as being 'parts of a whole'.

Students can define percentage as 'number of parts per hundred'.

Students are aware that percentages are used in everyday life.

KEYWORDS

Tier 2
Mixed, improper, terminating, increase, decrease, profit, loss, share, parts

Tier 3
Addition, subtraction, multiplication, division, fractions, recurring, reciprocal, integer, decimal, percentage, VAT, multiplier, ratio, proportion

4a. Fractions (N1, N2, N3, N10 ,N12, R3)	**Teaching time** 7–9 hours

OBJECTIVES

By the end of the sub-unit, students should be able to:

- Express a given number as a fraction of another;
- Find equivalent fractions and compare the size of fractions;
- Write a fraction in its simplest form, including using it to simplify a calculation, e.g. 50 ÷ 20 = = = 2.5;
- Find a fraction of a quantity or measurement, including within a context;
- Convert a fraction to a decimal to make a calculation easier;
- Convert between mixed numbers and improper fractions;
- Add, subtract, multiply and divide fractions;
- Multiply and divide fractions, including mixed numbers and whole numbers and vice versa;
- Add and subtract fractions, including mixed numbers;
- Understand and use unit fractions as multiplicative inverses;
- By writing the denominator in terms of its prime factors, decide whether fractions can be converted to recurring or terminating decimals;
- Convert a fraction to a recurring decimal;
- Convert a recurring decimal to a fraction;
- Find the reciprocal of an integer, decimal or fraction.

POSSIBLE SUCCESS CRITERIA/EXAM QUESTIONS

Express a given number as a fraction of another, including where the fraction is, for example, greater than 1, e.g. = = .

Answer the following: James delivers 56 newspapers. of the newspapers have a magazine. How many of the newspapers have a magazine?

Prove whether a fraction is terminating or recurring.

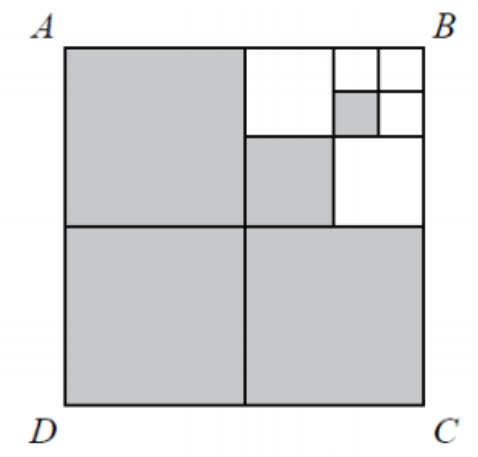
Convert a fraction to a decimal including where the fraction is greater than 1.

Prove algebraically that the recurring decimal $0.2\dot{5}$ has the value $\frac{23}{90}$

(Total 2 marks)

New SAMs Paper 2H qu.15 (N10 – AO1)

ABCD is a square.
This diagram is drawn accurately



What fraction of the square *ABCD* is shaded?

(Total 2 marks)
New SAMs Paper 2F qu.7 (N8 – AO1/AO2)

(*a*) Work out $\frac{2}{7}+\frac{1}{5}$

(2)

(*b*) Work out $1\frac{2}{3}\div\frac{3}{4}$

(2)

(Total 4 marks)
New SAMs Paper 1F qu.18 (N8 – AO1)

COMMON MISCONCEPTIONS

The larger the denominator, the larger the fraction.

NOTES

Ensure that you include fractions where only one of the denominators needs to be changed, in addition to where both need to be changed for addition and subtraction.

Include multiplying and dividing integers by fractions.

Use a calculator for changing fractions into decimals and look for patterns.

Recognise that every terminating decimal has its fraction with a 2 and/or 5 as a common factor in the denominator.

Use long division to illustrate recurring decimals.

Amounts of money should always be rounded to the nearest penny.

Encourage use of the fraction button.